



Development and identification of an analytical wake vortex model applying vortex line geometries derived from LES

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Operational Wake Vortex Models



Deutsches Zentrum
für Luft- und Raumfahrt e.V.
in der Helmholtz-Gemeinschaft

Content

- Introduction
- BIOT-SAVART & vortex line discretisation
- Vortex line geometry identification
- Validation by means of LES flow fields
- Conclusions and outlook



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Introduction

Wake vortex simulation models

- Aircraft simulation has proven in the past to be an effective tool for wake vortex encounter research in terms of
 - Cost
 - Reproducibility
 - Safety
- Depending on the purpose a variety of wake vortex models have been developed in the past
- Against the background of re-categorisation and revision of aircraft separations an important focus in simulation is on hazard assessment
- To gain realistic results for any kind of aircraft wake vortex encounter simulation modelling of wake is substantial

Introduction

Realistic vortex behaviour during decay

- The shape of wake vortices alters significantly during the decay
- Vortex deformation depends on atmospheric conditions
- Different states of vortex shape
 - Nearly straight
 - Sinusoidal shape due to Crow-instability
 - Linking
 - Vortex rings
- For realistic aircraft encounter simulation to be taken into account

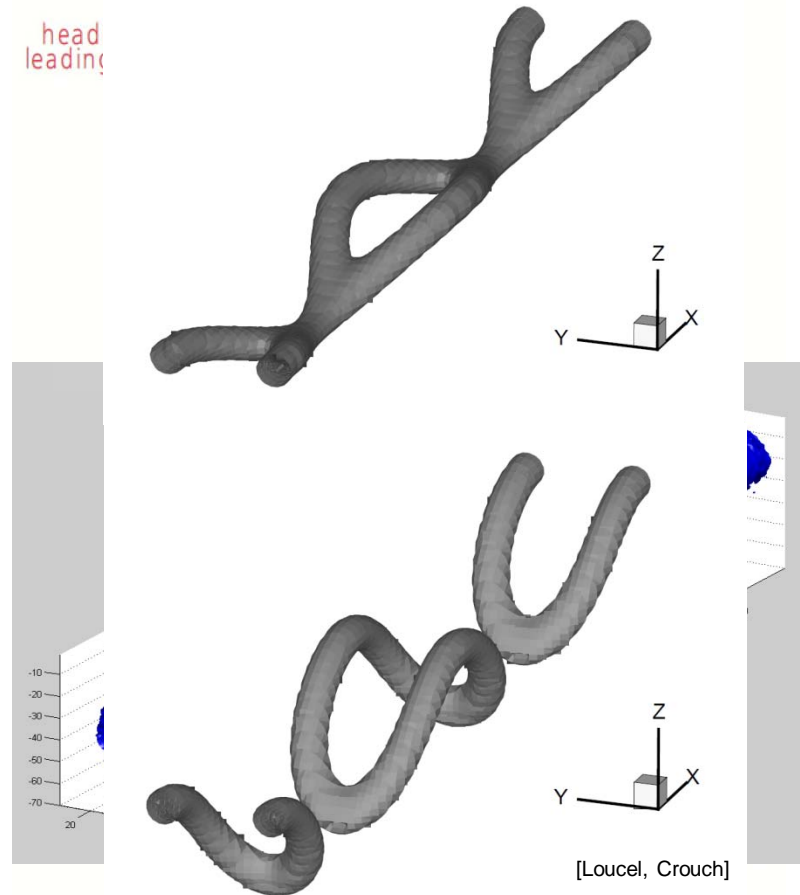


Introduction

Accuracy versus simplicity

- Analytical models assuming straight vortices
 - Advantage: low computational effort
 - Disadvantage: too simple for many applications (like hazard assessment)
- Very realistic numerical simulations (like Large-Eddy-Simulations)
 - Advantage: vortices most realistic during whole decay
 - Disadvantage: very high computational effort for real-time aircraft simulation
- Compromise: combination of both!

Is it possible to make it even more realistic with the same model simplicity?

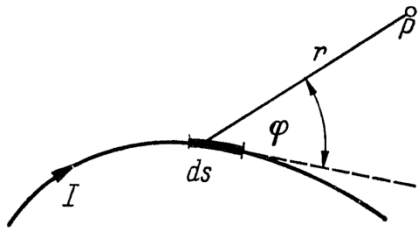


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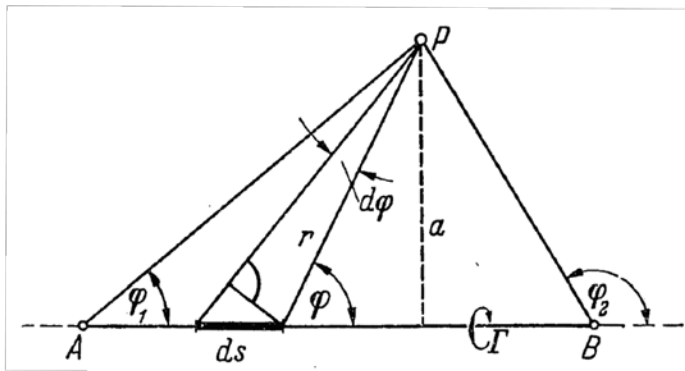
The BIOT-SAVART-law

- In analogy to electro dynamics vortex flow field calculated with BIOT-SAVART-law



$$w = \frac{\Gamma}{4\pi} \oint \frac{\sin \varphi}{r^2} ds$$

- BIOT-SAVART-law can be solved analytically for straight vortex lines



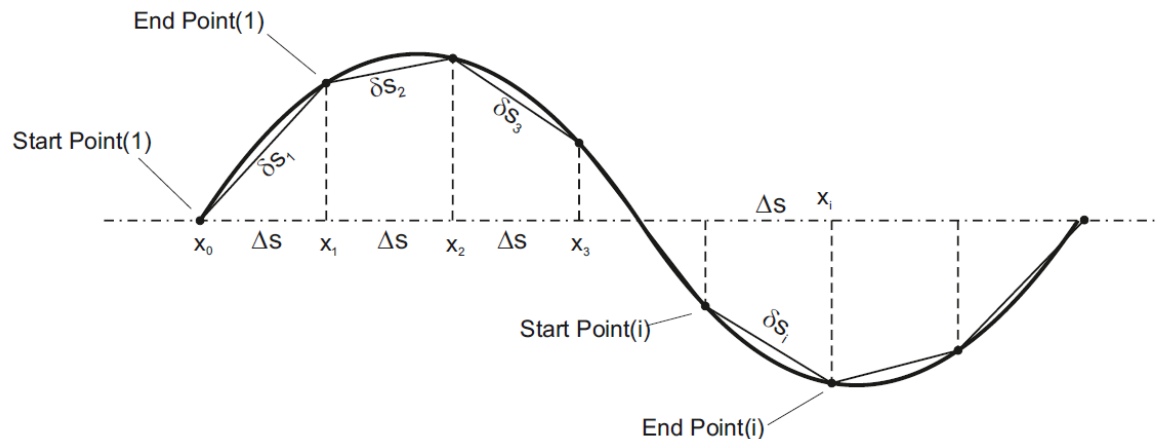
vortex segment AB

$$w = \frac{\Gamma}{4\pi a} \int_{\varphi_1}^{\varphi_2} \sin \varphi d\varphi$$

$$w = \frac{\Gamma}{4\pi a} (\cos \varphi_1 - \cos \varphi_2)$$

Vortex line discretisation

- Arbitrary wake vortex line discretised into N straight vortex segments



- Vortex flow field is sum of induced velocities of all N wake elements (in this case with Burnham-Hallock radial velocity distribution)

$$\varpi = \sum_{i=1}^N \frac{\Gamma}{4\pi} \frac{a_i}{a_i^2 + r_c^2} (\cos \varphi_{1i} - \cos \varphi_{2i})$$

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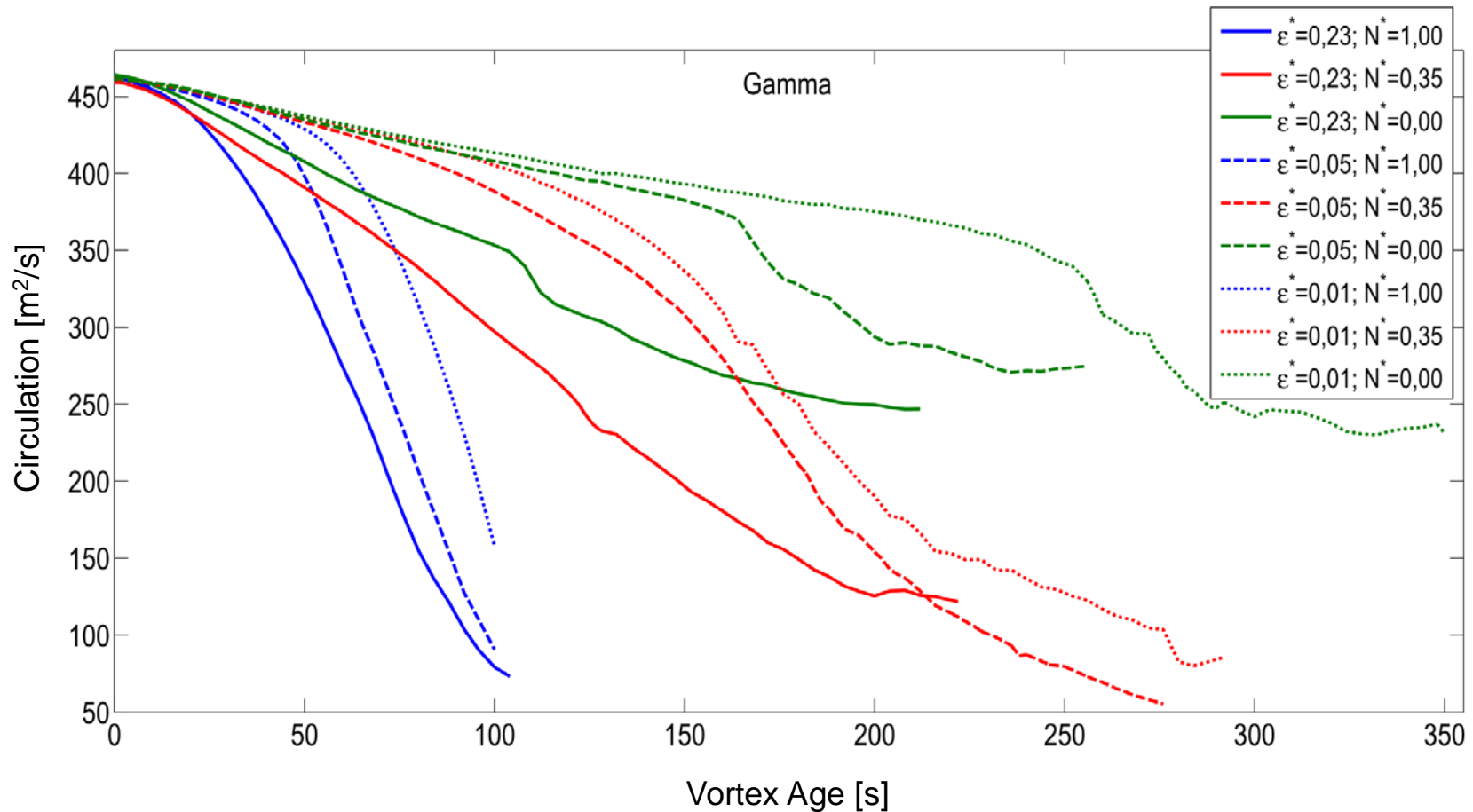
Geometry identification

- Different mathematical approaches for wavy vortices and vortex rings necessary
- Identification based on vortex line geometries derived from LES
 - Generator aircraft A340-300 in landing configuration
 - 9 different vortex evolutions (geometry and flow field) as a function of atmospheric conditions in terms of turbulence and temperature stratification
 - No to moderate turbulence / neutral to stable temperature stratification
 - Geometry datasets in 2-4 s steps covering the whole decay

-22.192	-26.001	0.104	-511.719	462.071	0.33	0.36	0.30	3.11	17.07	0.00
-22.195	-28.979	0.114	-488.806	462.629	-0.63	0.19	0.27	3.01	16.75	-0.01
-22.229	-32.011	0.149	-441.502	462.211	-0.65	0.65	0.22	2.76	16.79	0.00
-22.261	-35.029	0.200	-450.372	461.562	-0.60	0.97	0.16	3.39	16.64	0.00
-22.281	-38.001	0.199	-484.042	460.810	-0.39	-0.02	0.14	3.21	17.11	-0.01
-22.256	-40.964	0.116	-434.590	460.851	0.48	-1.60	0.17	3.25	16.47	0.00
-22.226	-43.967	0.043	-436.993	461.833	0.57	-1.40	0.18	2.95	16.62	0.00
-22.229	-47.013	0.021	-474.388	461.989	-0.04	-0.41	0.15	3.28	17.12	0.00
-22.272	-50.053	0.094	-475.426	460.755	-0.82	1.38	-0.01	2.87	16.42	0.00
-22.252	-53.023	0.208	-456.406	462.151	0.39	2.21	-0.25	3.19	16.41	-0.01
-22.221	-55.967	0.219	-463.066	460.106	0.60	0.20	-0.38	3.13	16.78	0.00
-22.198	-58.925	0.175	-456.762	462.132	0.43	-0.84	-0.40	3.01	17.18	0.00

Geometry identification

LES vortices



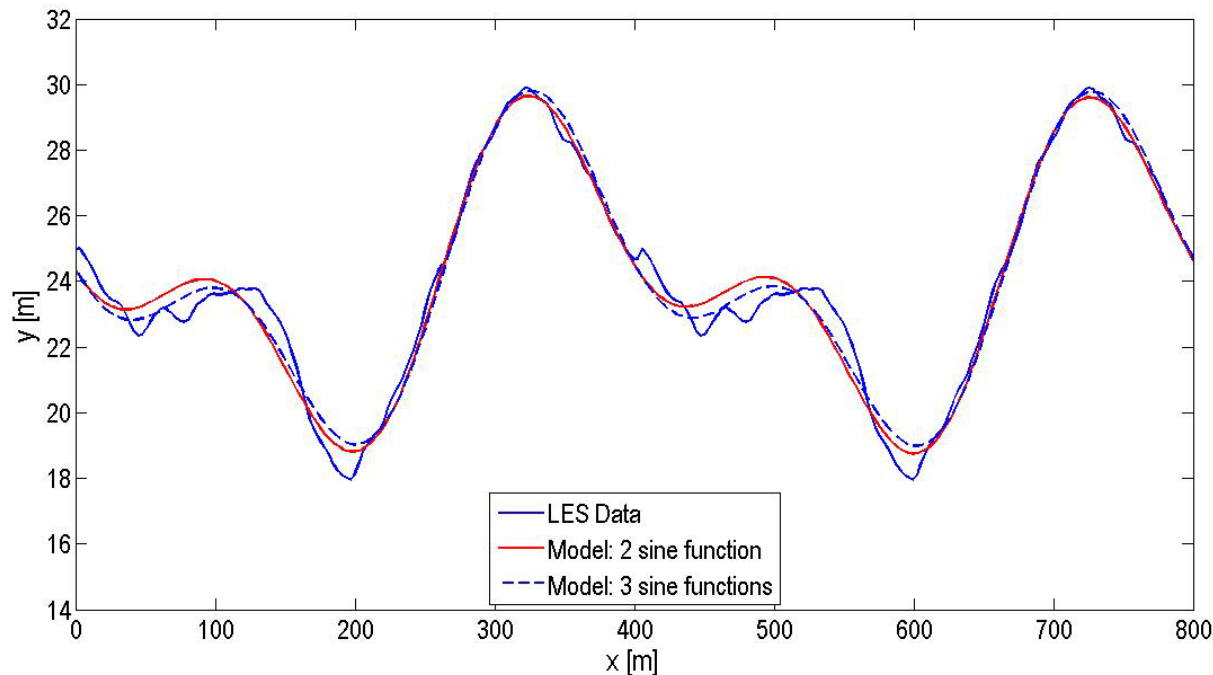
Geometry identification

Wavy vortices

➤ 2 sine function approach

$$y = p_{1y} \cdot \sin(p_{2y} \cdot x + p_{3y}) + p_{4y} \cdot \sin(p_{5y} \cdot x + p_{6y}) + p_{7y}$$

$$z = p_{1z} \cdot \sin(p_{2z} \cdot x + p_{3z}) + p_{4z} \cdot \sin(p_{5z} \cdot x + p_{6z}) + p_{7z}$$



Geometry identification

Vortex rings

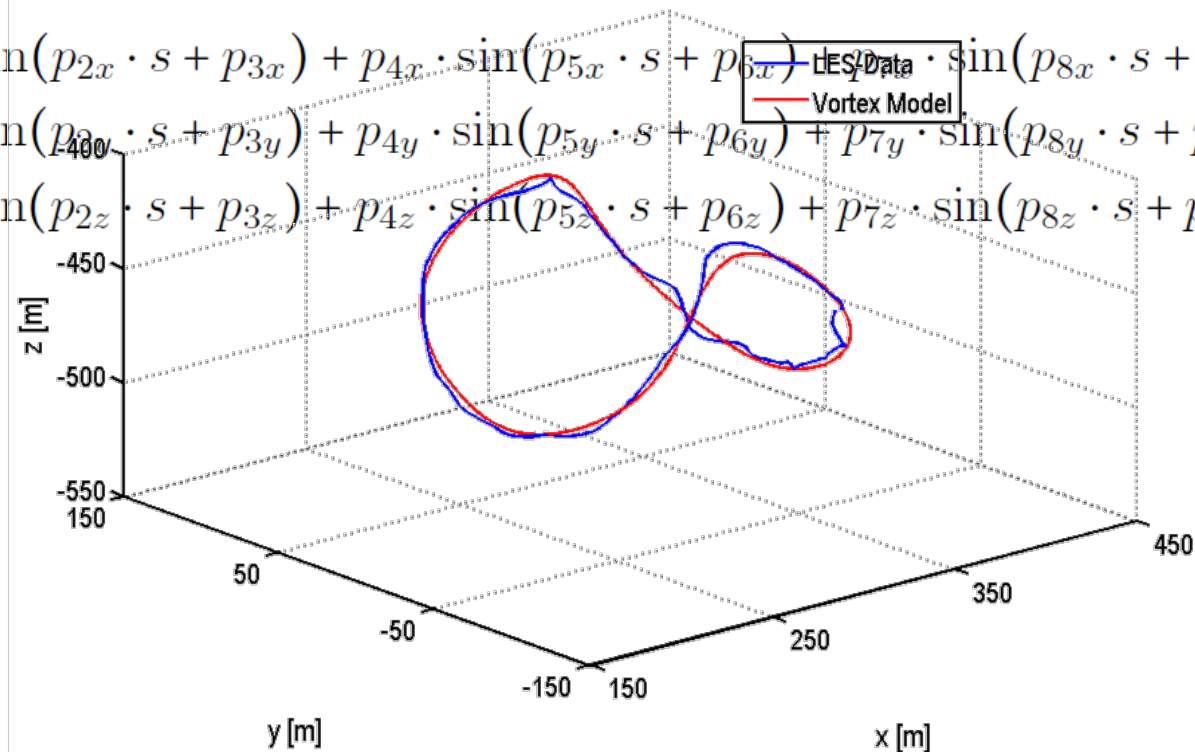
➤ 3 sine function approach

➤ Discretisation variable $s \in [0, 2\pi]$ $t = 320 \text{ s}$

$$x = p_{1x} \cdot \sin(p_{2x} \cdot s + p_{3x}) + p_{4x} \cdot \sin(p_{5x} \cdot s + p_{6x}) + p_{7x} \cdot \sin(p_{8x} \cdot s + p_{9x}) + p_{10x}$$

$$y = p_{1y} \cdot \sin(p_{2y} \cdot s + p_{3y}) + p_{4y} \cdot \sin(p_{5y} \cdot s + p_{6y}) + p_{7y} \cdot \sin(p_{8y} \cdot s + p_{9y}) + p_{10y}$$

$$z = p_{1z} \cdot \sin(p_{2z} \cdot s + p_{3z}) + p_{4z} \cdot \sin(p_{5z} \cdot s + p_{6z}) + p_{7z} \cdot \sin(p_{8z} \cdot s + p_{9z}) + p_{10z}$$

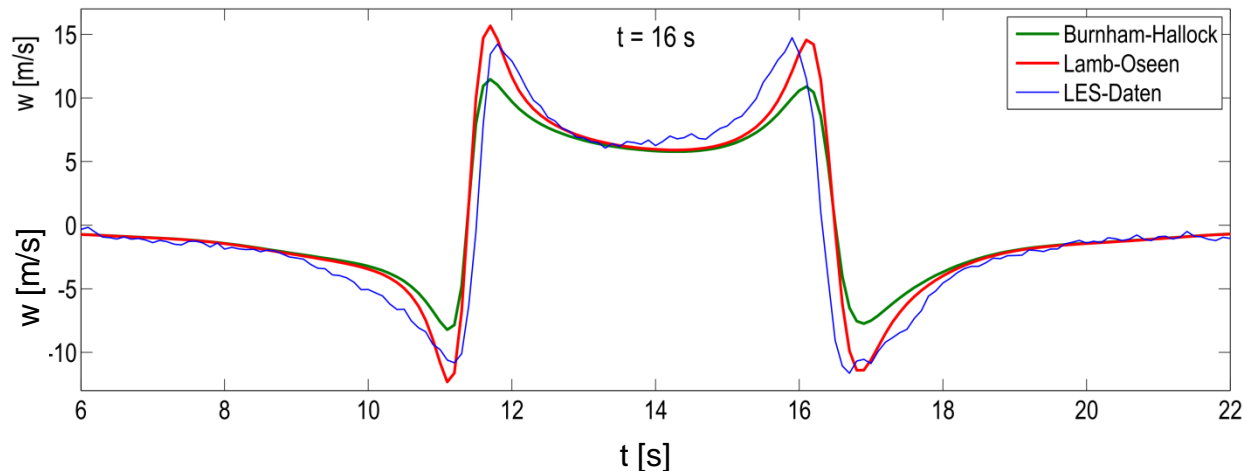


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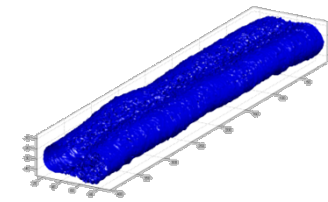
Validation

- Identification by means of LES vortex line geometry
- Validation by means of LES flow fields
- Evolution of circulation, core radius, etc. mean values from LES
- Comparison of velocity components on predefined paths through the flow fields
- For radial velocity distribution two models compared
 - Burnham-Hallock
 - Lamb-Oseen

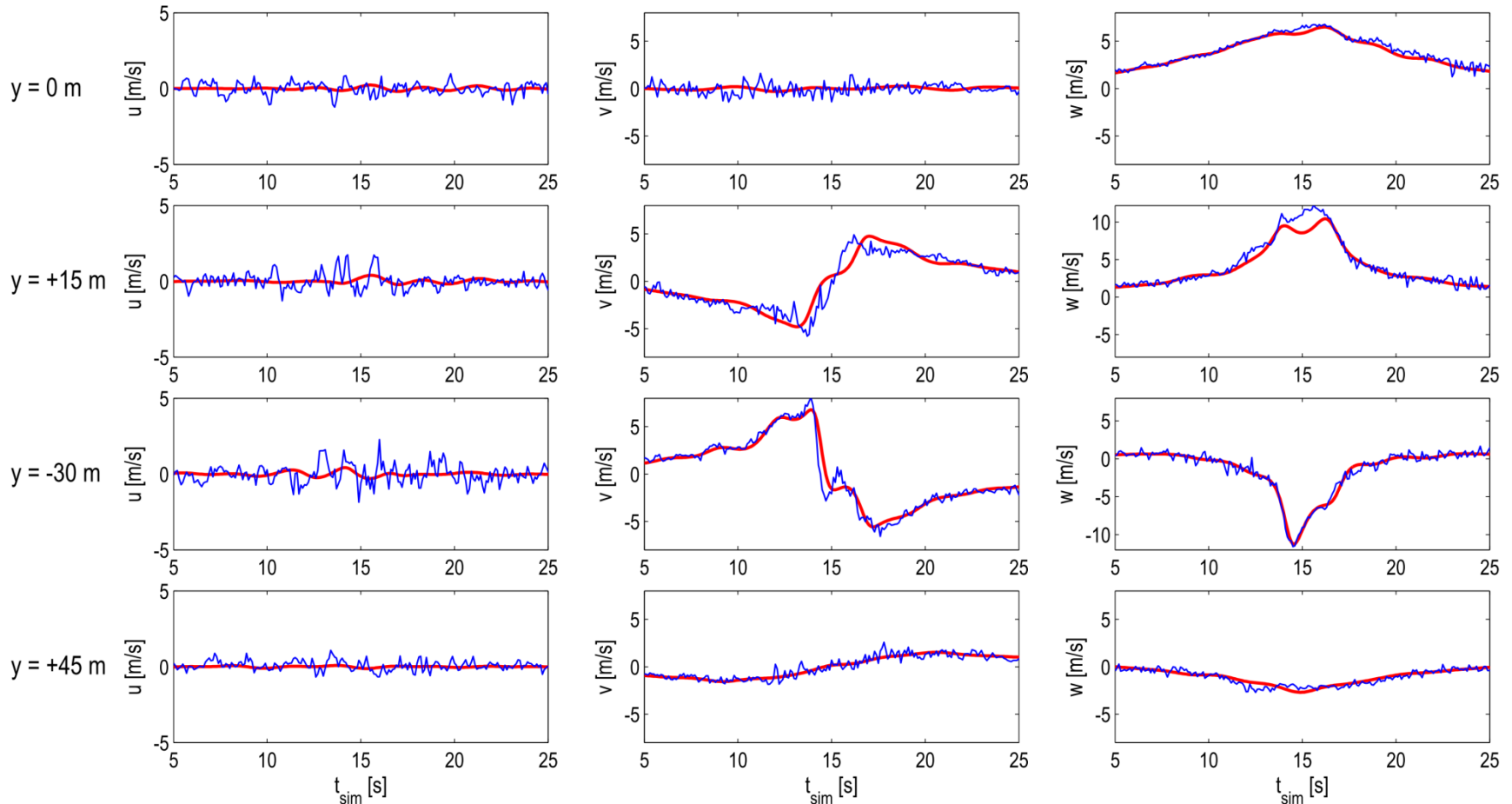


Validation

Nearly straight vortices

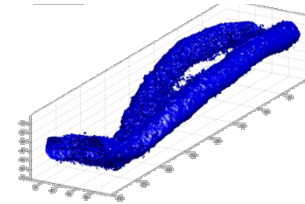


$t = 16 \text{ s}$

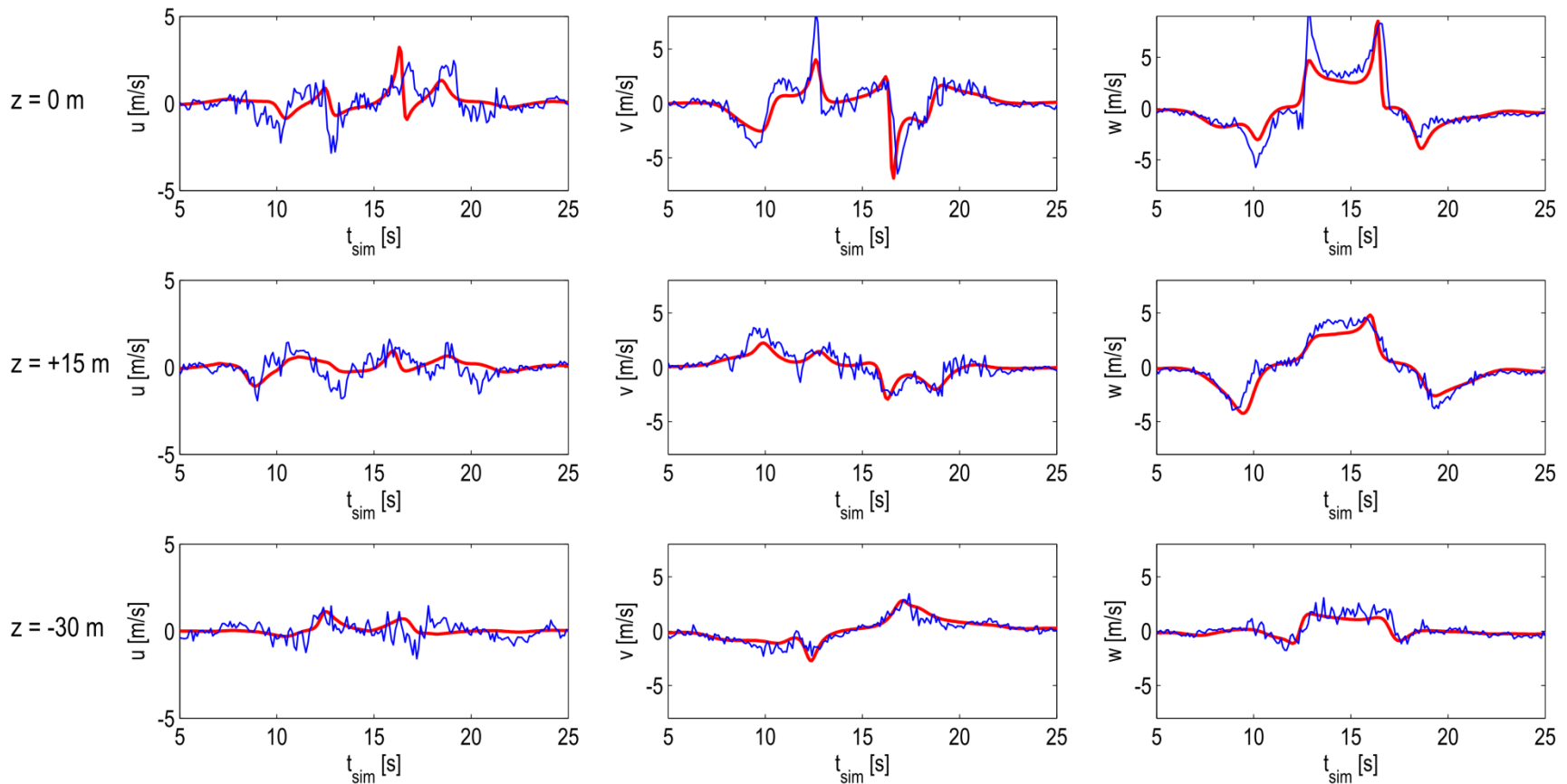


Validation

Wavy Vortices

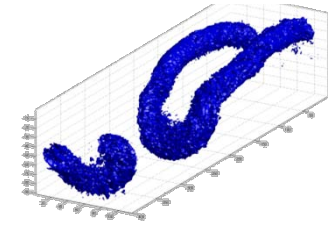


$t = 108 \text{ s}$

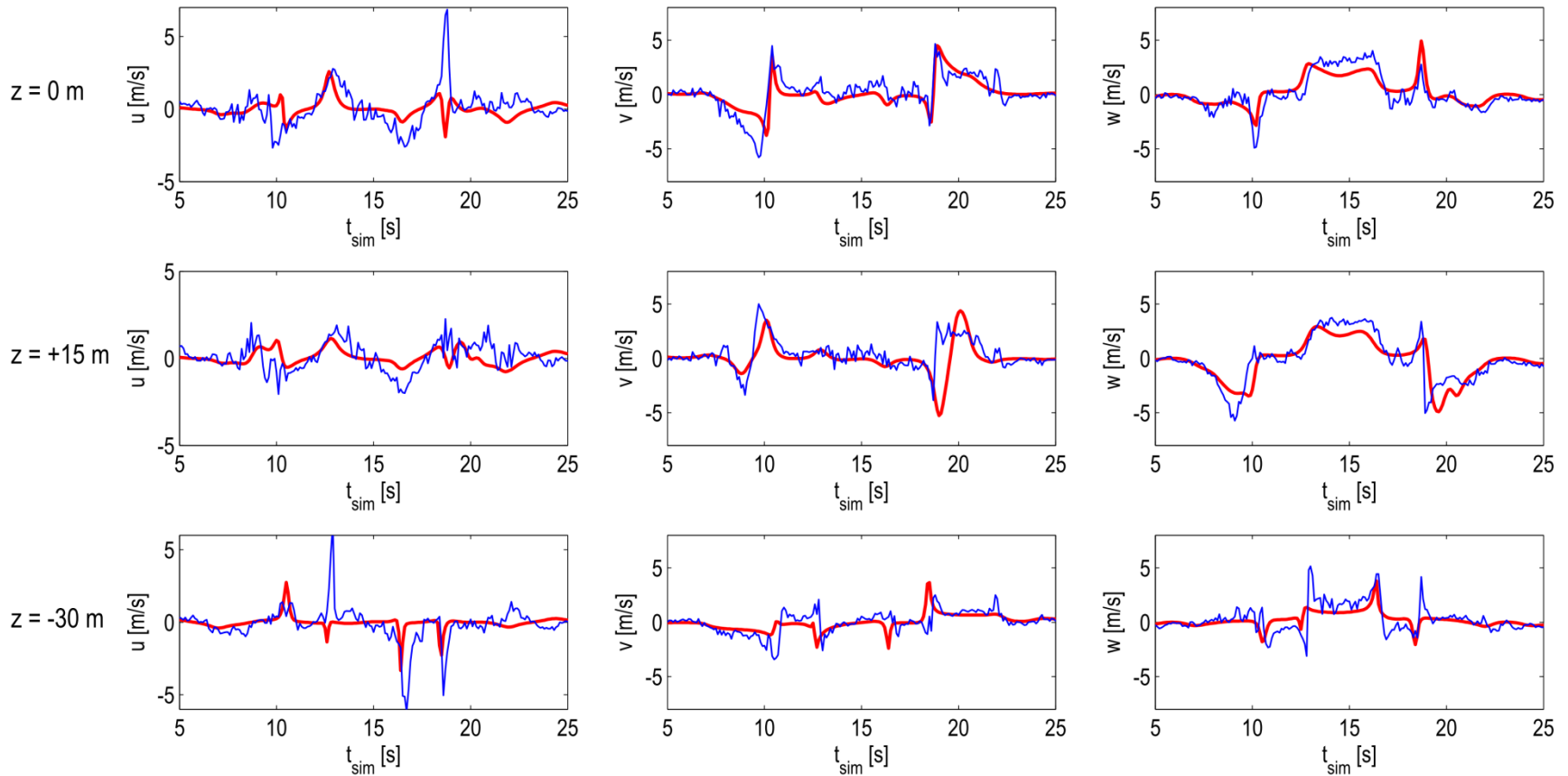


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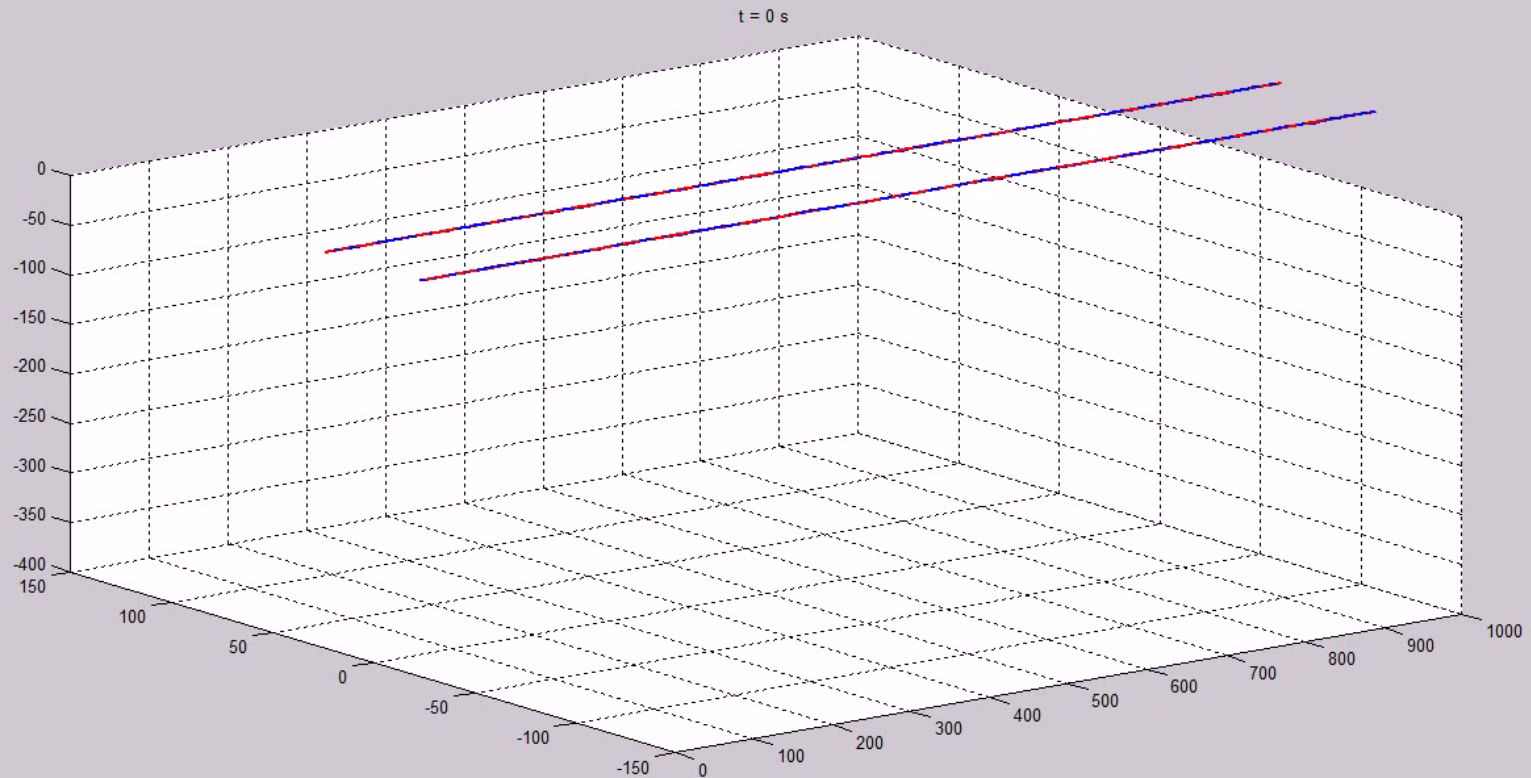
Vortex Rings



$t = 136 \text{ s}$



Validation



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Conclusions

- Analytical wake vortex model applying arbitrarily shaped vortex line geometries
- Two different approaches for wavy vortices and vortex rings
- Parameter identification to adjust model geometry to LES vortex line geometries
- LES results for 9 different atmospheric conditions
- Validation by means of LES flow fields
- Evolution of circulation, core radius, etc. mean values from LES
- Model matches LES results very well in terms of geometry and flow field!
- Model still solely applicable to generator aircraft A340 in landing configuration

Outlook

- Investigation on scalability of the model (other aircraft than A340)
- Investigation on interpolation of atmospheric conditions
- Usage of model for
 - Aircraft simulation with fixed wake vortices (fixed in space and time)
 - Dynamic wake simulation with moving vortices
 - Combination of vortex model with traffic server of DLR simulator
 - Ability of testing wake alleviation or avoidance systems under realistic operational conditions
 - Simulation of realistic encounters



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Thank you for your attention...



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