WakeNet3-Europe Specific Workshop
« RE-CATEGORIZATION »
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Aircraft wake vortices: physics and UCL models

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Developed wakes, shortly after rollup

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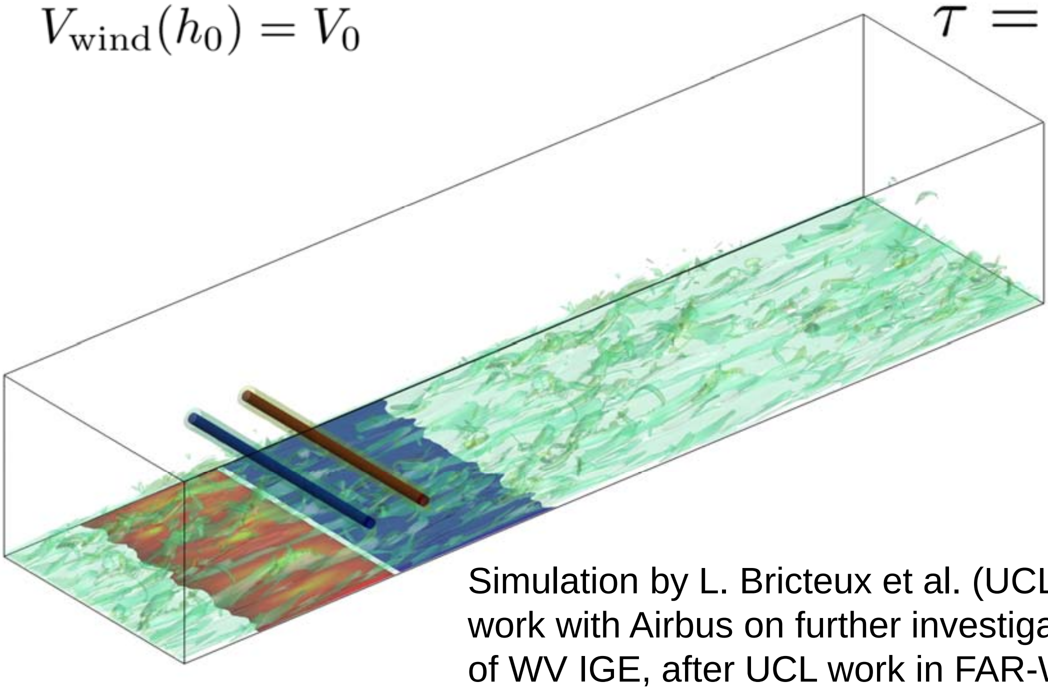
2VS IGE generated close to the ground and with weak cross wind

$$h_0 = b_0$$

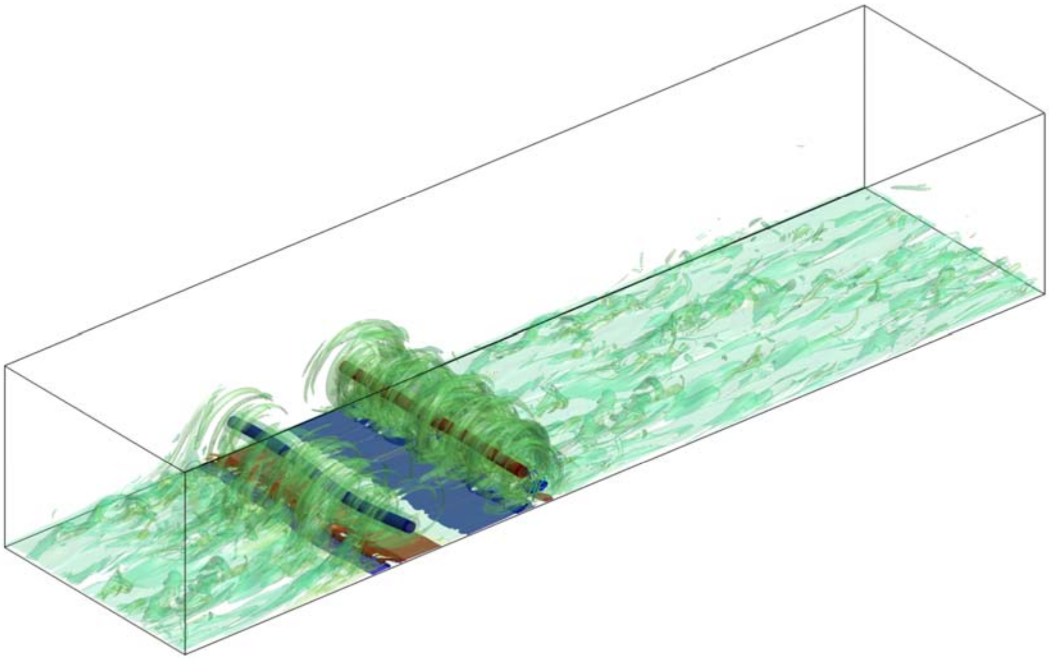
$$V_{\mathrm{wind}}(h_0) = V_0$$

$$\tau = 0.0$$



Simulation by L. Bricteux et al. (UCL) work with Airbus on further investigations of WV IGE, after UCL work in FAR-Wake

$\tau = 1.28$



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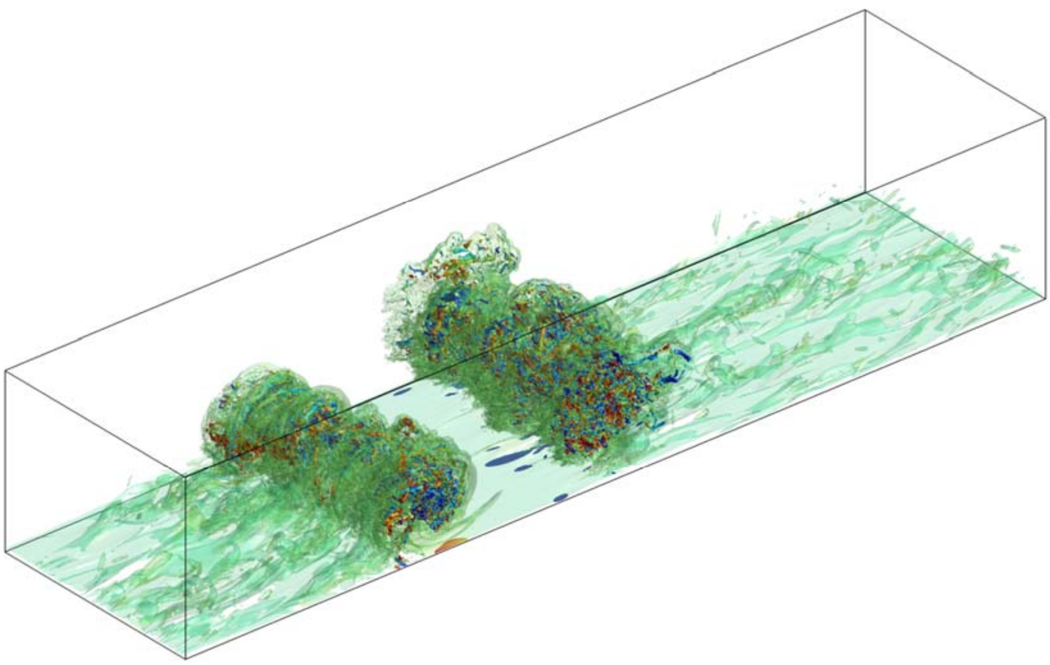
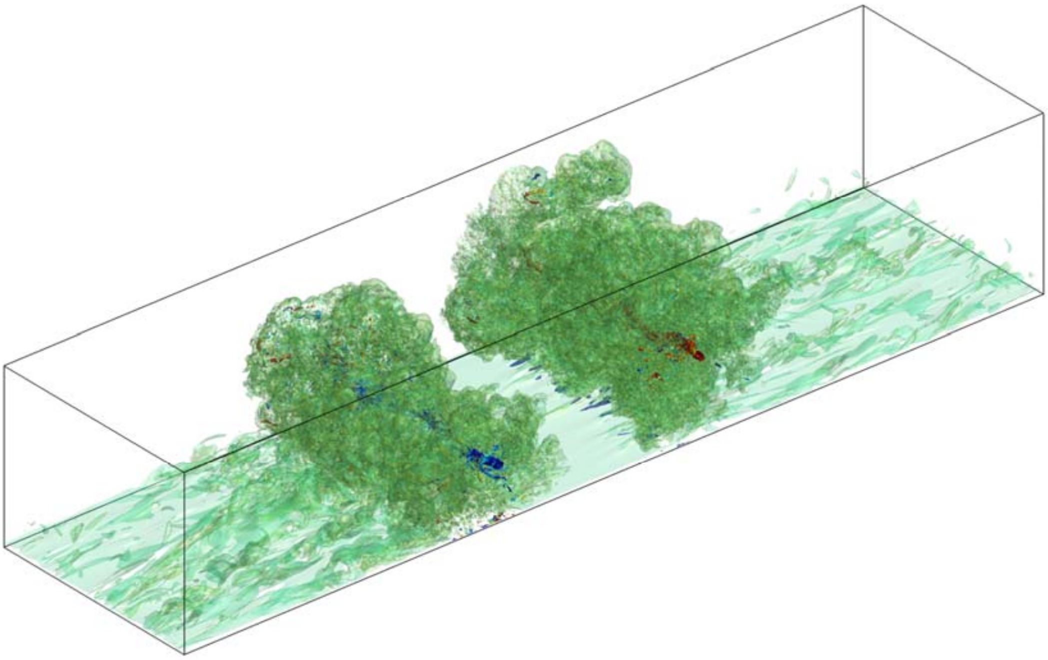
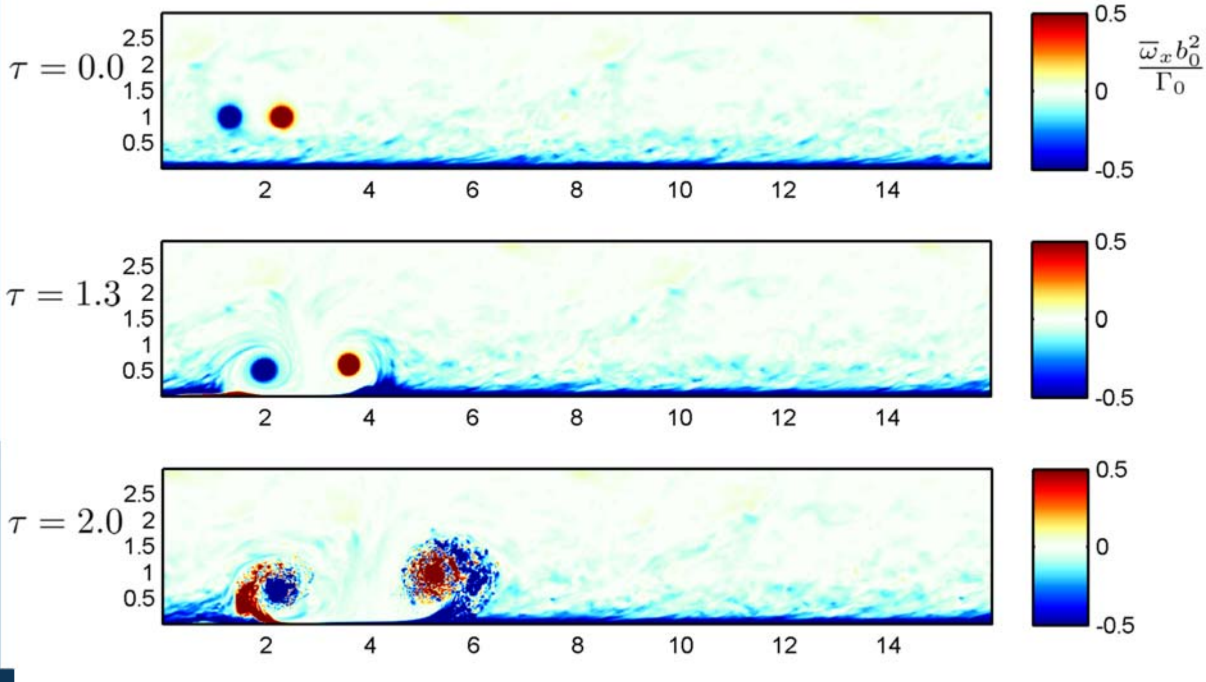
$\tau = 2.56$



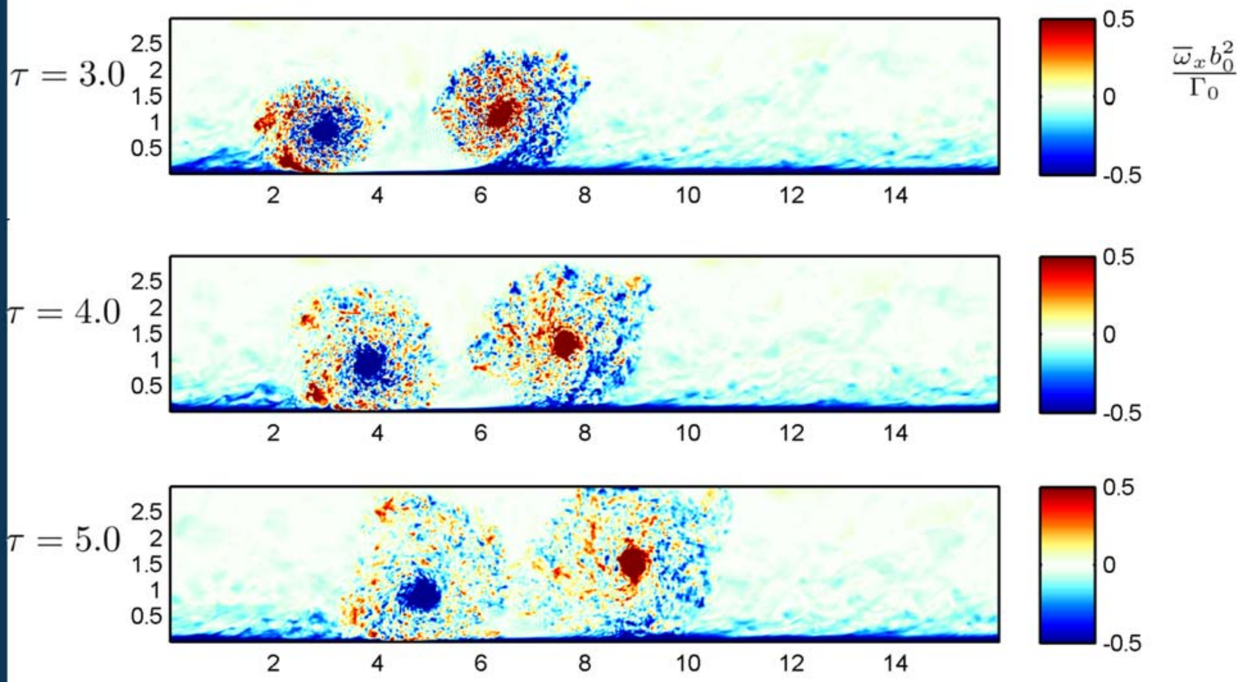
$$\tau = 3.84$$



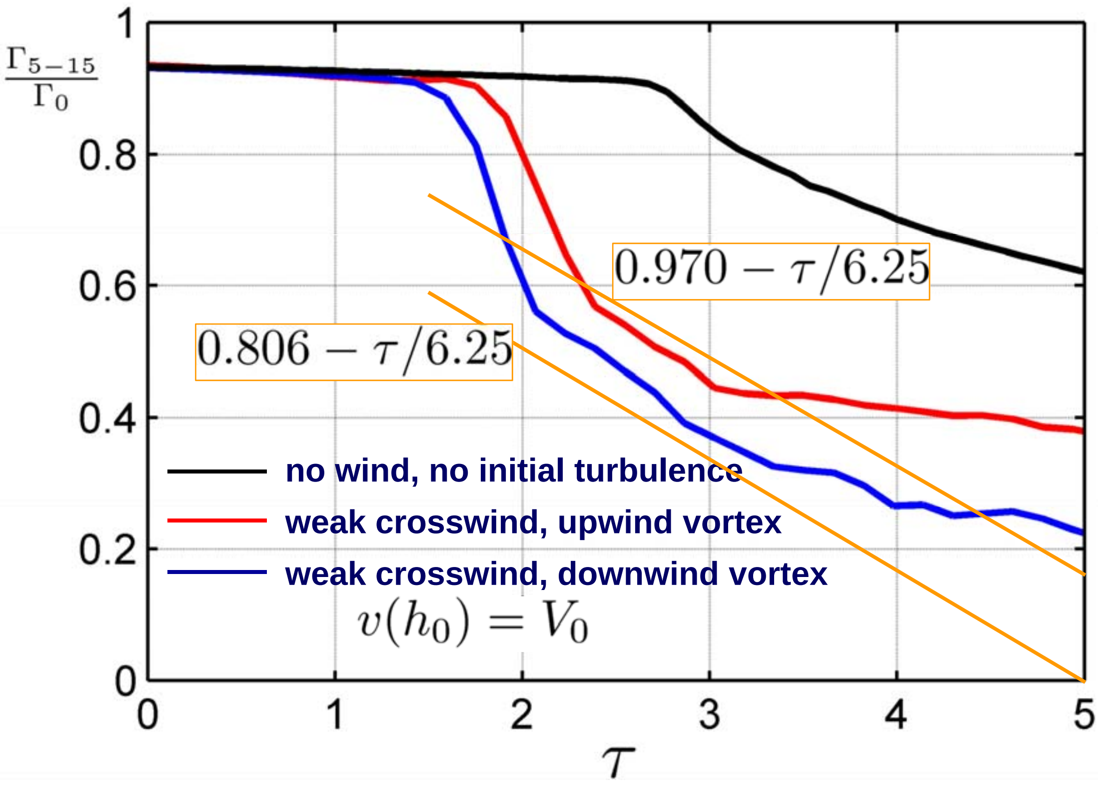
Longitudinally averaged flow



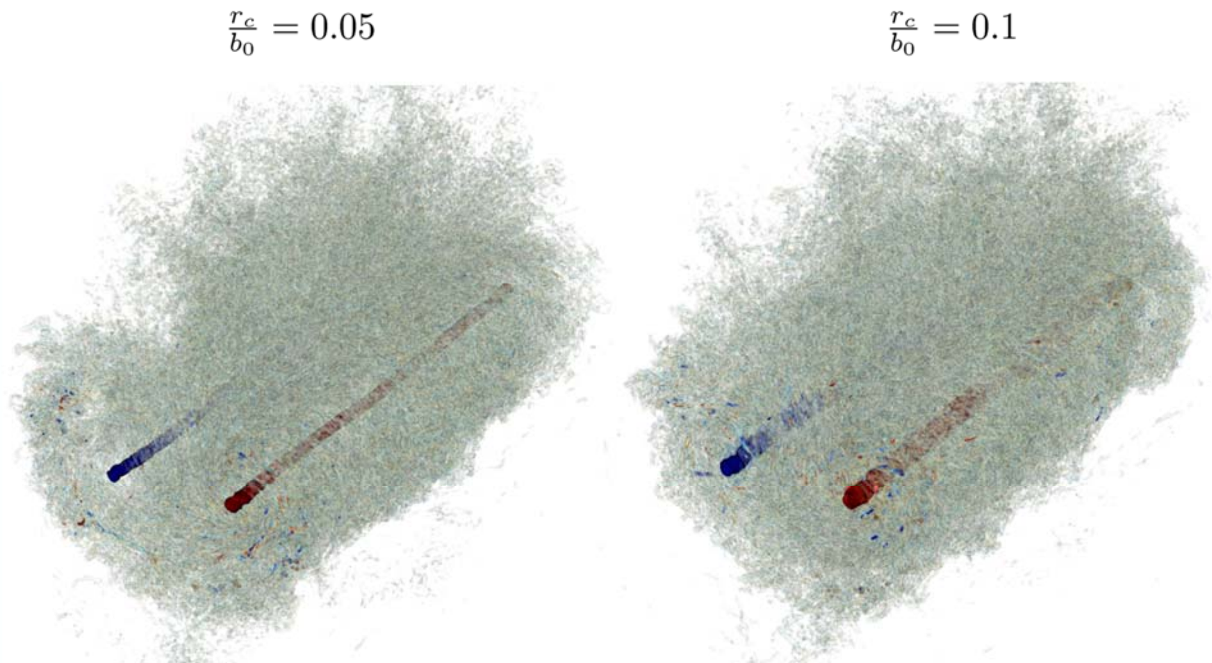
Longitudinally averaged flow



Circulation decay (RECAT I model also shown)



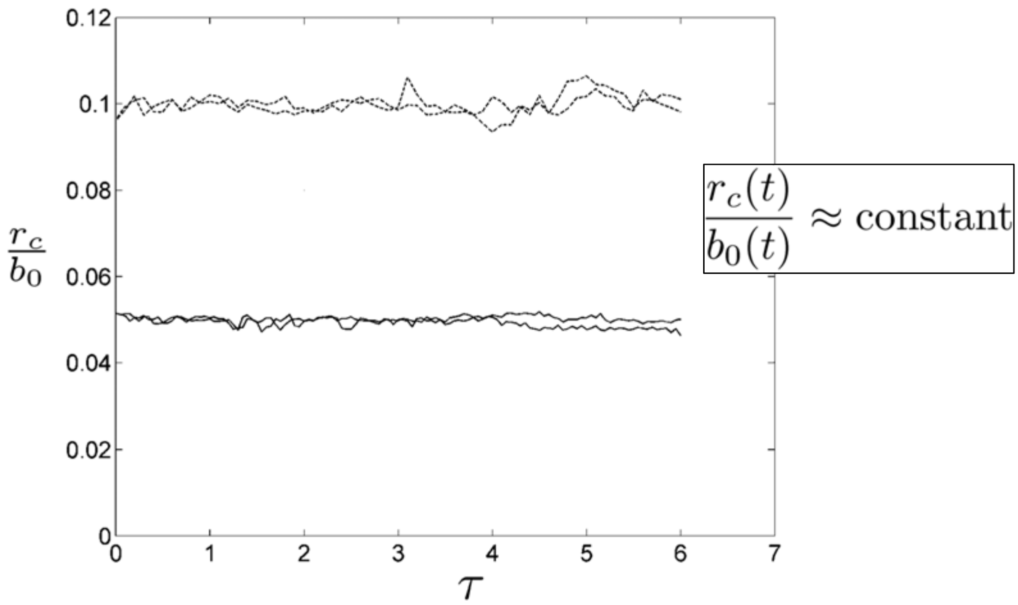
2VS OGE and without Crow instabilities



Iso-λ_2 surfaces colored by the axial vorticity

Cases with same initial circulation but different core sizes: thus energy and induced drag are different. Source: De Visscher et al. (UCL)

Time evolution of the core radius:

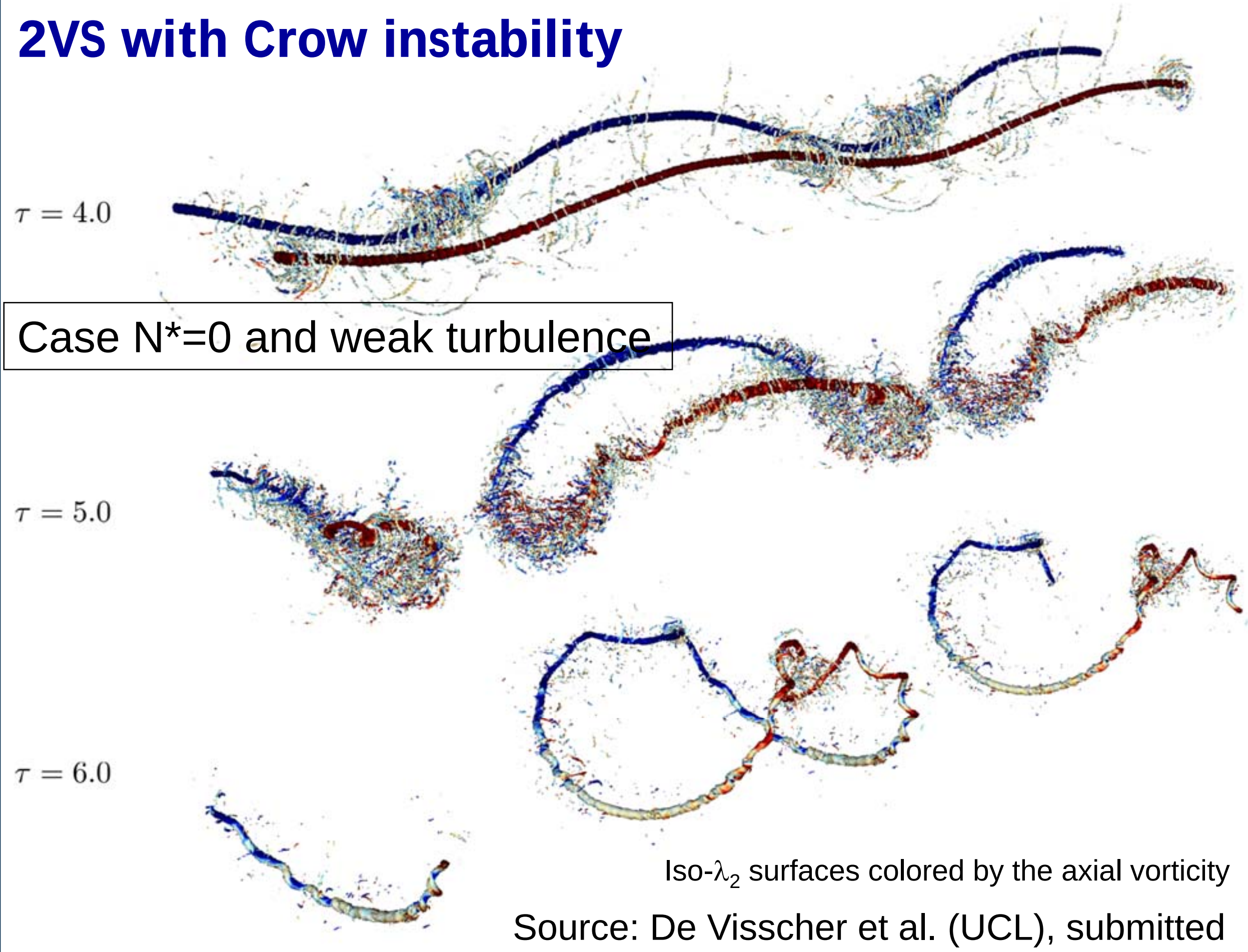


——— Case 1: $\frac{r_c}{b_0} = 0.05$

.... Case 2: $\frac{r_c}{b_0} = 0.1$

For each case, the curve presents the average of the core radius measurements.
Those were measured in each cross-plane, and "sitting" on each local vortex center (port and starboard)

2VS with Crow instability

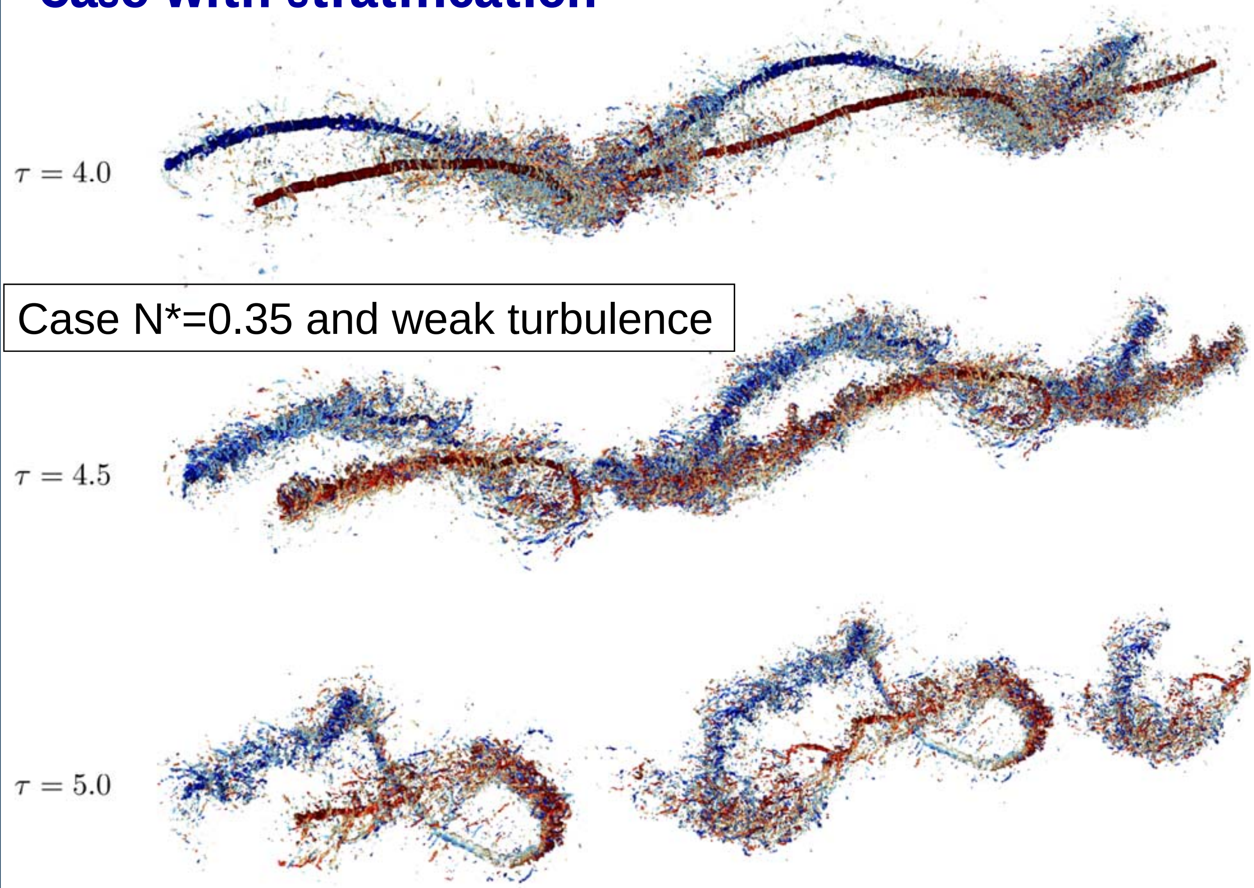


Case N*=0 and weak turbulence

Iso-λ_2 surfaces colored by the axial vorticity

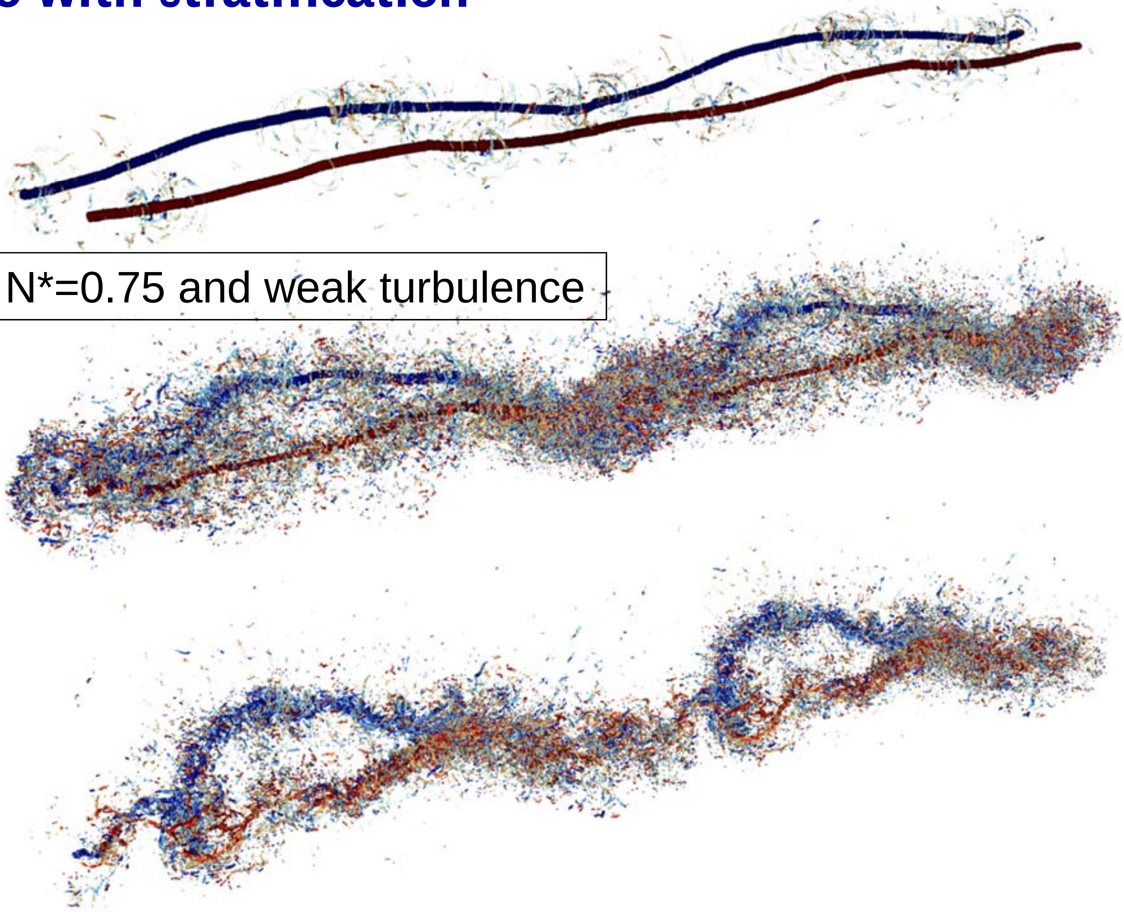
Source: De Visscher et al. (UCL), submitted

Case with stratification



Case N*=0.35 and weak turbulence

Case with stratification



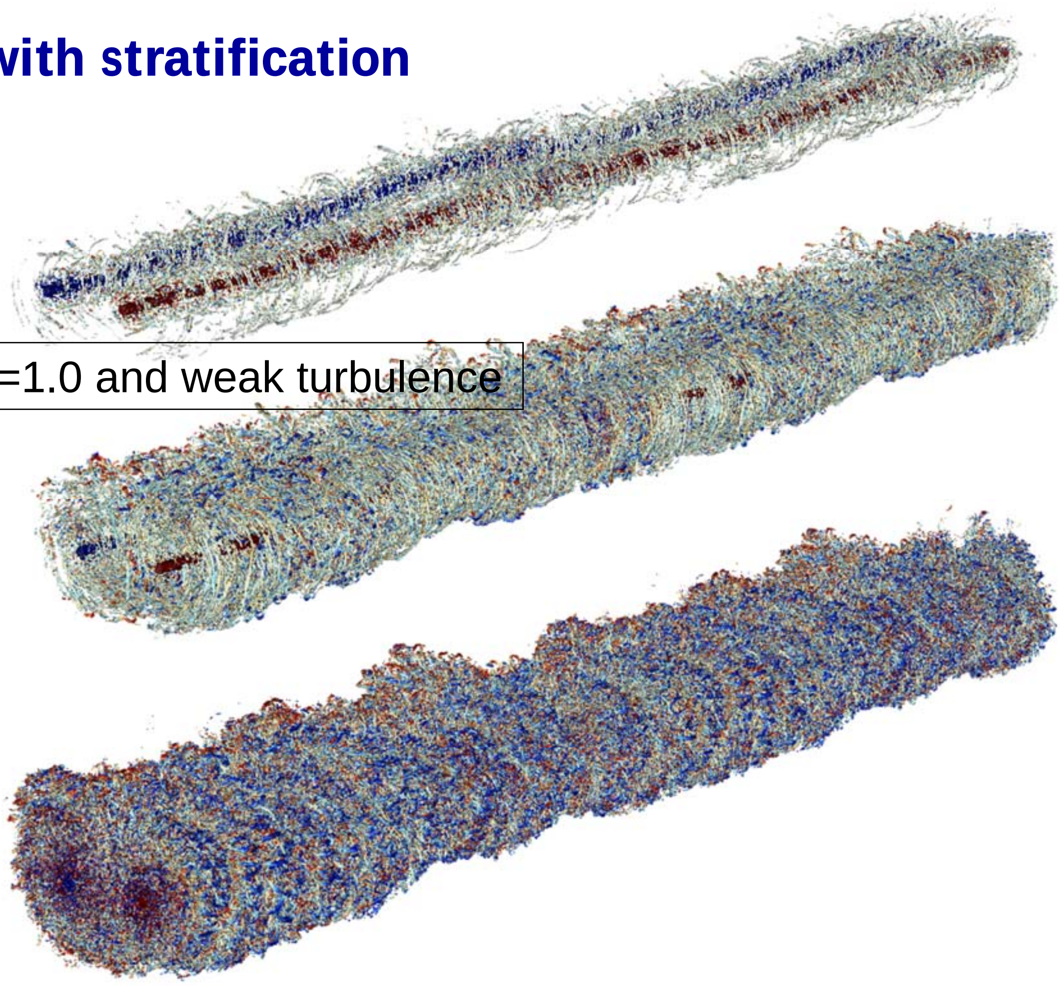
$\tau = 2.0$

Case N*=0.75 and weak turbulence

$\tau = 3.0$

$\tau = 3.5$

Case with stratification



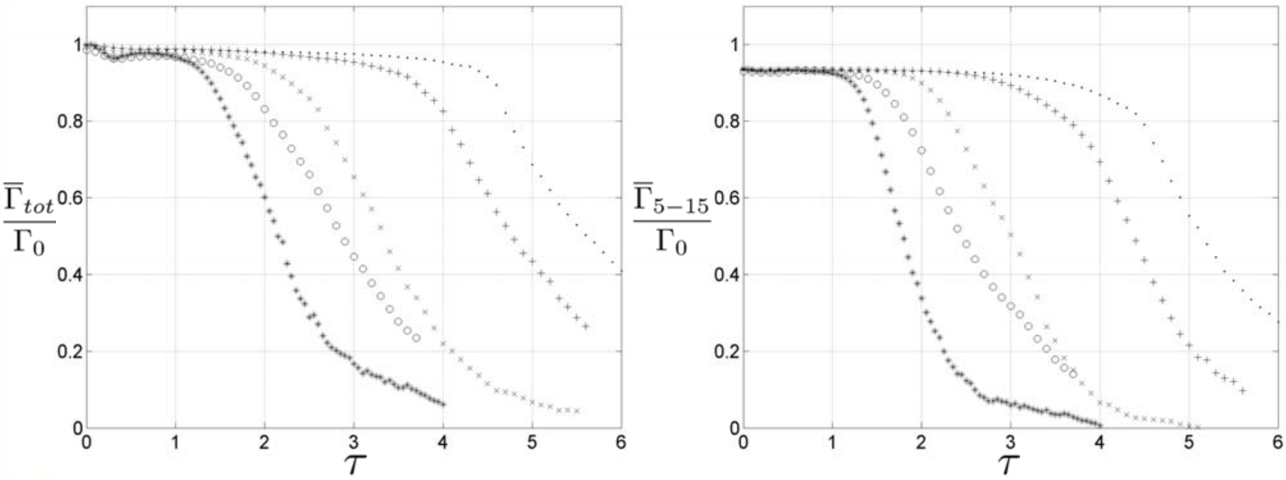
$\tau = 1.0$

Case N*=1.0 and weak turbulence

$\tau = 1.5$

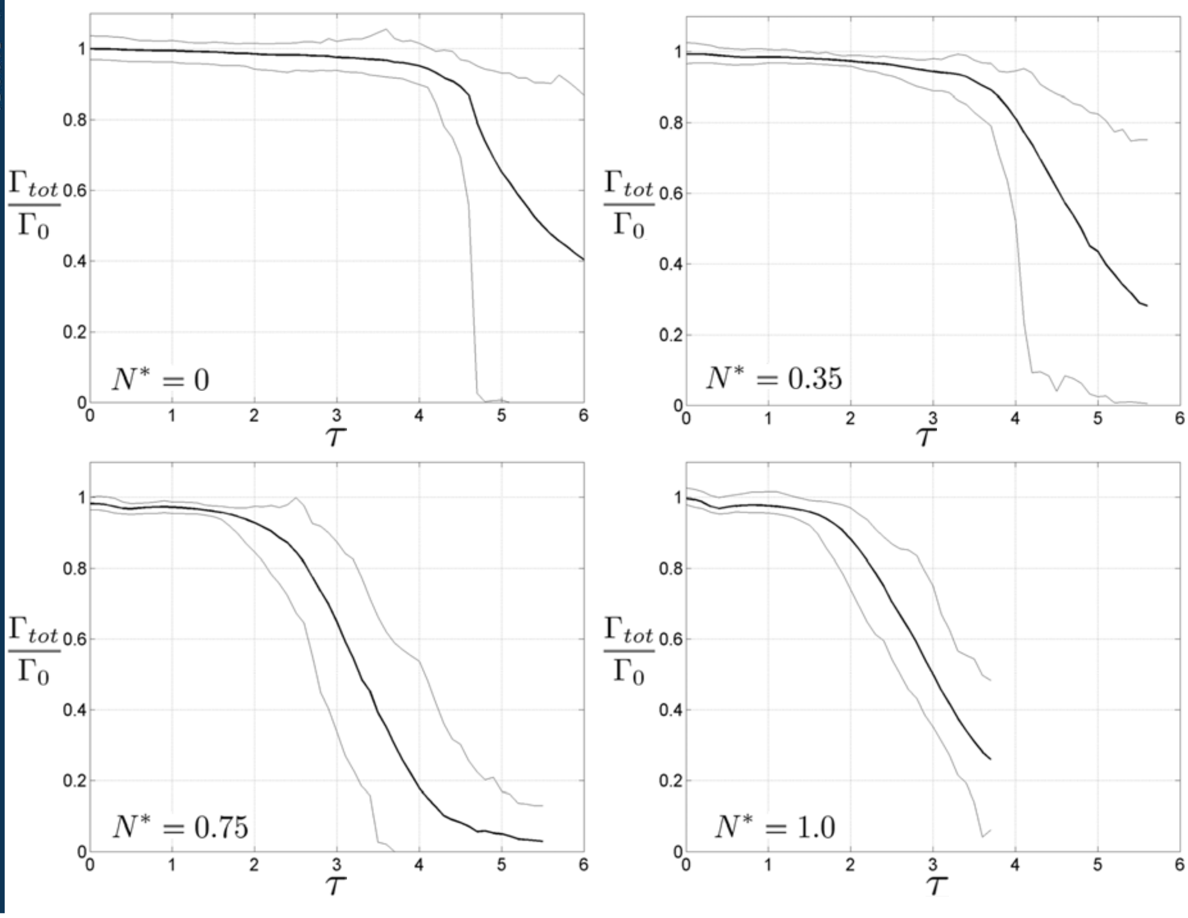
$\tau = 2.0$

Comparison of the longitudinally-averaged circulation evolution



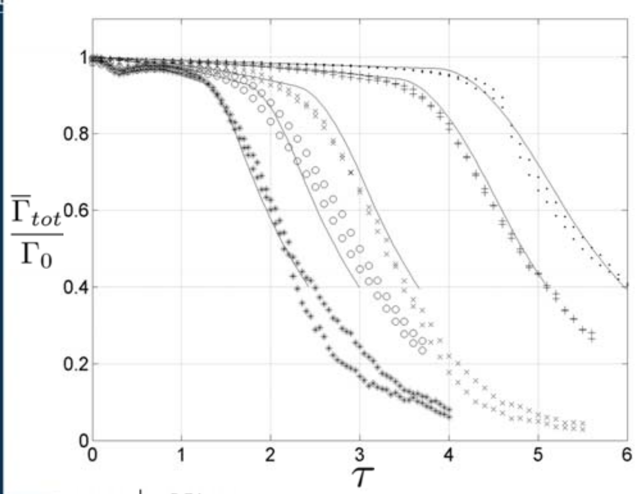
- · $N^* = 0$
- $+$ $N^* = 0.35$
- $\times$ $N^* = 0.75$
- ○ $N^* = 1.0$
- $*$ $N^* = 1.4$

Comparison of the mean and 95%-envelope circulation evolution



Improved circulation decay modeling

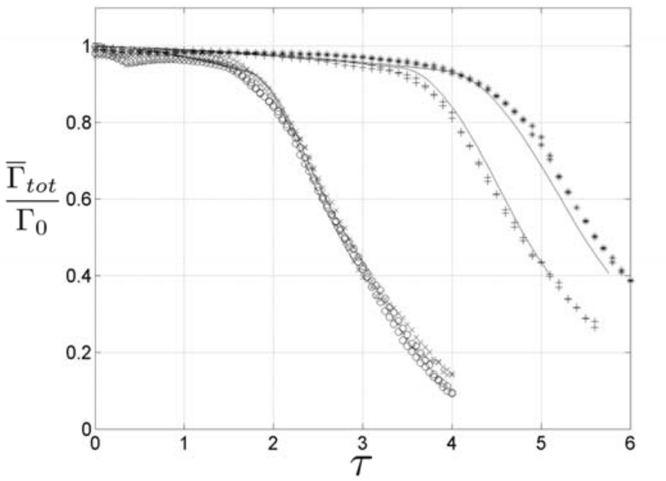
Stratification effect



	N^*
$\cdot$	0
$+$	0.35
$\times$	0.75
$\circ$	1.0
$*$	1.4

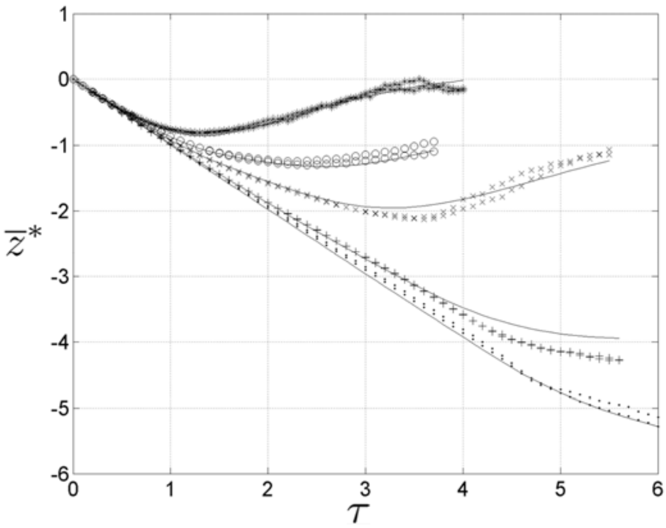
$$\epsilon^* = \frac{\epsilon\, b_0}{V_0^3} = 2.42\, 10^{-4}$$

Stratification and turbulence effects



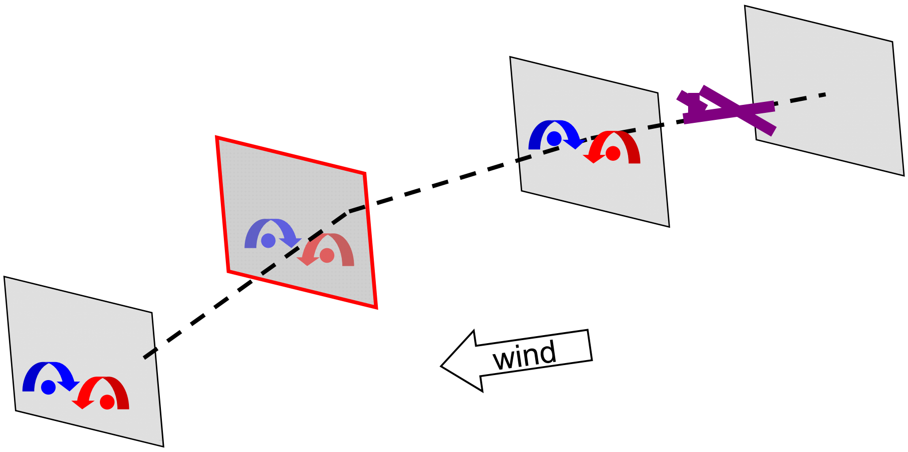
	N^*	ϵ^*
$*$	0.35	$2.42\, 10^{-5}$
$+$	0.35	$2.42\, 10^{-4}$
$\times$	1.0	$2.42\, 10^{-5}$
$\circ$	1.0	$2.42\, 10^{-4}$

Improved vortex transport modeling



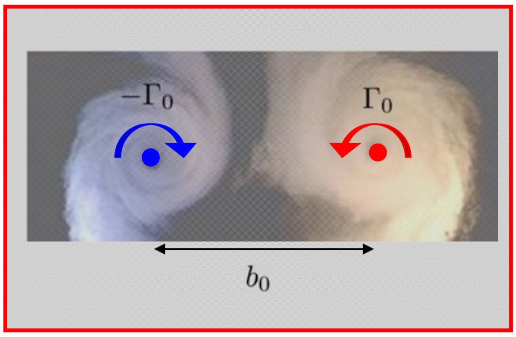
WAKE4D: "3-D space + time" wake vortex prediction platform

- It uses as input:
  - the a/c trajectory
  - the met conditions.
- The computational domain is divided in various "computational gates" crossing the flight path.
- The aircraft crossing one of the gates generates a pair of wake vortices (WV) that are transported (also by the headwind) and also decay.



The Deterministic wake Vortex Model (DVM)

- The DVM forecasts, in real-time, the WV behavior (transport and decay) in one computational gate, using simplified physical models.
- The initial wake is computed using the a/c characteristics (position, mass, TAS, wingspan, lift distribution, and flight angles) when the aircraft crosses the gate.



$$
\begin{aligned}
b_0 &= s\,b \\
\Gamma_0 &= \frac{Mg}{\rho U_\infty b_0} \\
V_0 &= \frac{\Gamma_0}{2\pi b_0} \\
t_0 &= \frac{b_0}{V_0}
\end{aligned}
$$

- Each primary wake vortex is represented by a vortex particle with a chosen circulation distribution profile:
  - Burnham-Hallock model (= low-order algebraic model),
  - high-order algebraic model, or
  - two-scales Proctor-Winckelmans model
- Alternative: wake vortex sheet model discretized using small vortex particles

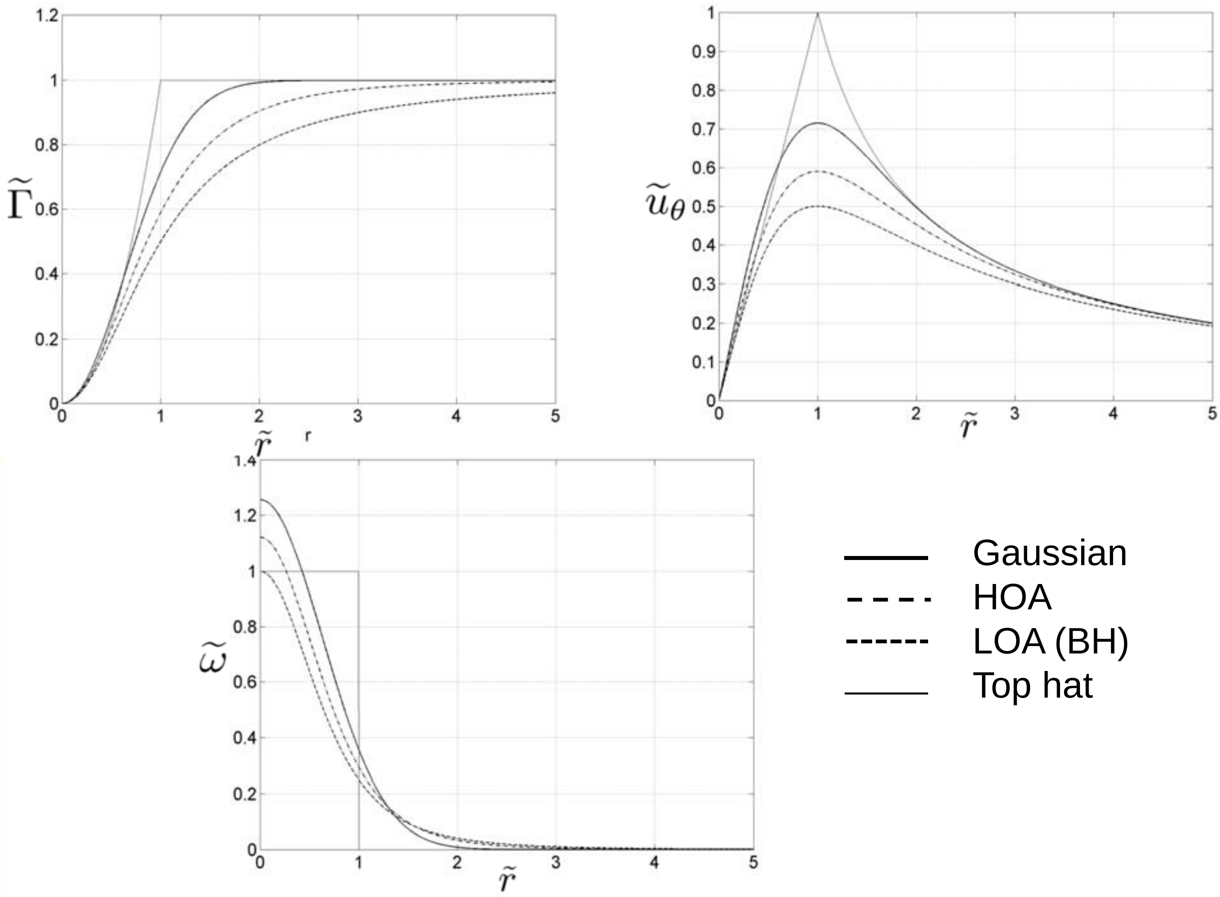
“One-scale” models for each vortex of the 2VS

- Definitions :

$$
\begin{aligned}
u_\theta(r_c) &= \max\left(u_\theta(r)\right) \\
\Rightarrow \tilde{r} &= r/r_c
\end{aligned}
\qquad
\begin{aligned}
\widetilde{\Gamma} &= \Gamma/\Gamma_0 \\
\widetilde{u}_\theta &= \frac{u_\theta}{\Gamma_0/(2\pi\, r_c)} \\
\widetilde{\omega} &= \frac{\omega}{\Gamma_0/(\pi\, r_c^2)}
\end{aligned}
$$

	$\widetilde{\Gamma}\,(\tilde{r})$	$\widetilde{u}_\theta\,(\tilde{r})$	$\widetilde{\omega}\,(\tilde{r})$
Gaussian ($\beta = 1.256$)	$1 - \exp\left(-\beta\,\tilde{r}^2\right)$	$\frac{1}{\tilde{r}}\left(1 - \exp\left(-\beta\,\tilde{r}^2\right)\right)$	$\beta \exp\left(-\beta\,\tilde{r}^2\right)$
Low Order Algebraic	$\frac{\tilde{r}^2}{(\tilde{r}^2+1)}$	$\frac{\tilde{r}}{(\tilde{r}^2+1)}$	$\frac{1}{(\tilde{r}^2+1)^2}$
High Order Algebraic ($\gamma = 1.781$)	$\frac{\tilde{r}^2\left(\tilde{r}^2+2\,\gamma\right)}{(\tilde{r}^2+\gamma)^2}$	$\frac{\tilde{r}\left(\tilde{r}^2+2\,\gamma\right)}{(\tilde{r}^2+\gamma)^2}$	$\frac{2\,\gamma^2}{(\tilde{r}^2+\gamma)^3}$
Top Hat $0 \leq \tilde{r} < 1$	$\tilde{r}^2$	$\tilde{r}$	1
$\tilde{r} \geq 1$	1	$\frac{1}{\tilde{r}}$	0

“One-scale” models for each vortex of the 2VS

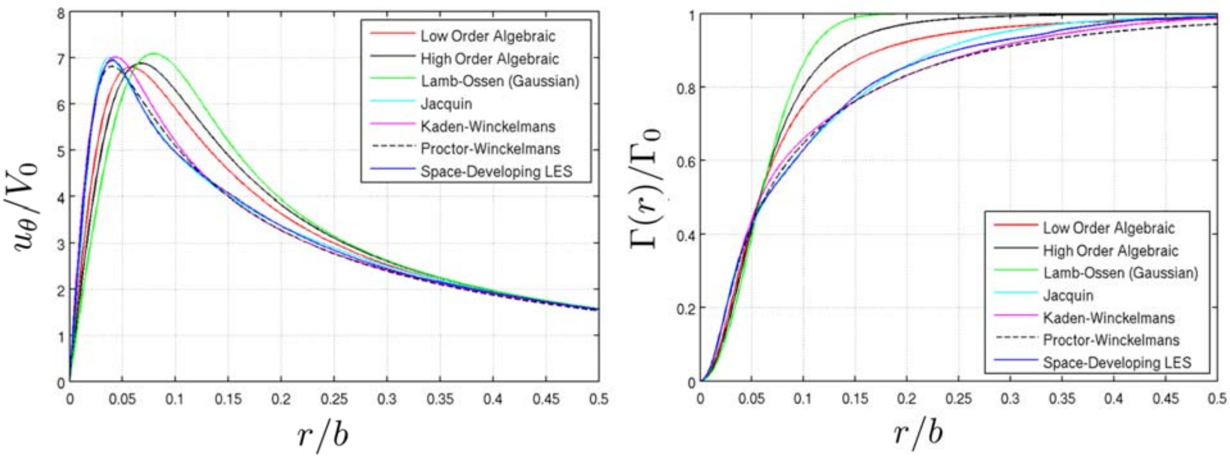


“Two-scales” models for each vortex of the 2VS

- e.g., Jacquin, Proctor-Winckelmans
- Proctor-Winckelmans model: $\frac{\Gamma(r)}{\Gamma_0} = 1 - \exp\left(- \frac{\beta_i \left(\frac{r}{b}\right)^2}{\left(1 + \left(\frac{\beta_i}{\beta_o}\left(\frac{r}{b}\right)^{5/4}\right)^p\right)^{1/p}} \right)$

  with $\beta_o = 10, p = 3 \ldots 5$, and β_o / β_i determined by r_c / b

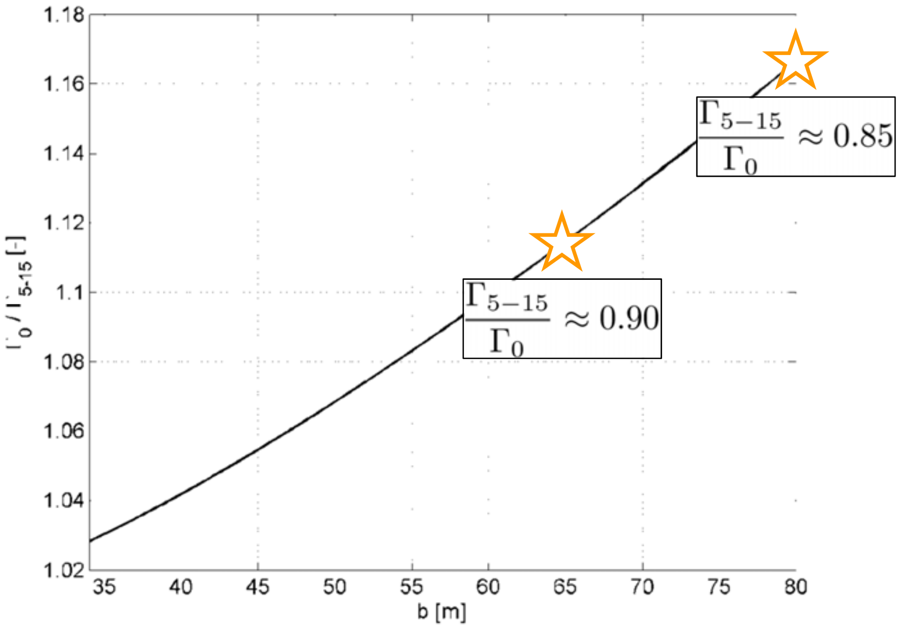
- Two-scales models are superior in term of azimuthal velocity and circulation profiles (here calibration from space-developing simulation of WV rollup)



Jackson et al., TP 13629E, 2001
de Bruin and Winckelmans, AW-114-25, Awiator, 2005
Lonfils et al., TR1.1.2-6, Far-Wake, 2008

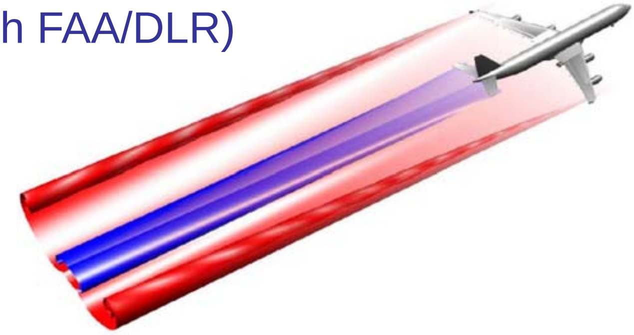
Circulations: Γ_0 versus Γ_{5-15}

- The WV total circulation Γ_0 is greater than the “Lidar processed” WV circulation $\Gamma_{5\text{-}15}$.
- Model of the initial ratio Γ_0 / $\Gamma_{5\text{-}15}$ as a function of the wingspan b:

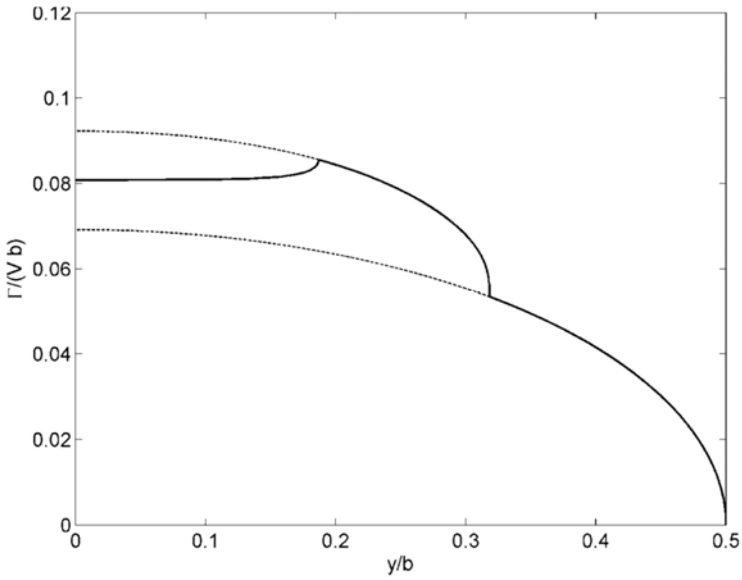


Source: UCL presentation at WN3E 1st major workshop

Vortex sheet model for Near-Field WVE analysis (also collaboration with TUB and with FAA/DLR)

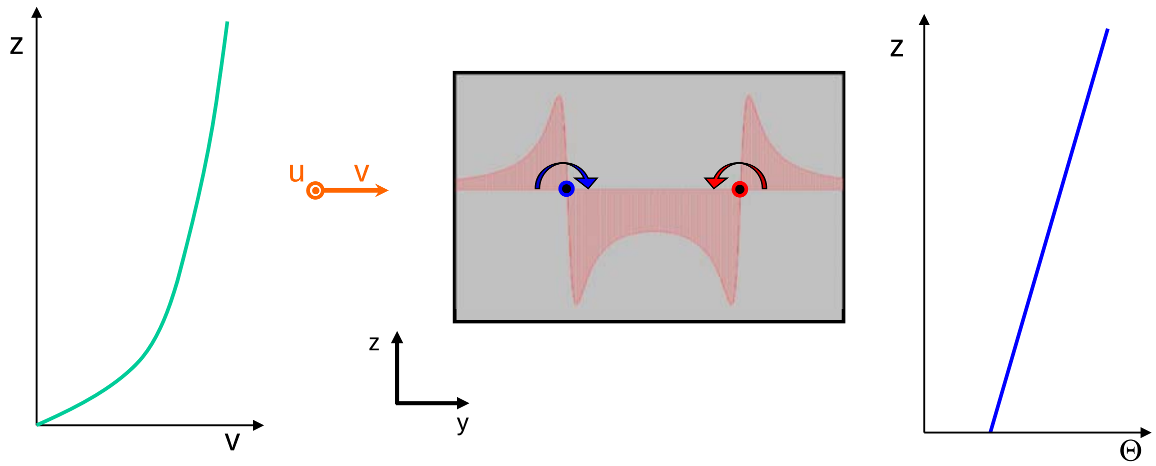


- Parametric model taking into account :
  - the tip vortex,
  - the outer flap vortex, and possibly
  - the HTP vortex
- The roll-up is calculated using the DVM with several particles
- A routine calculating the near-wake induced velocity field was also developed and used by FAA/DLR



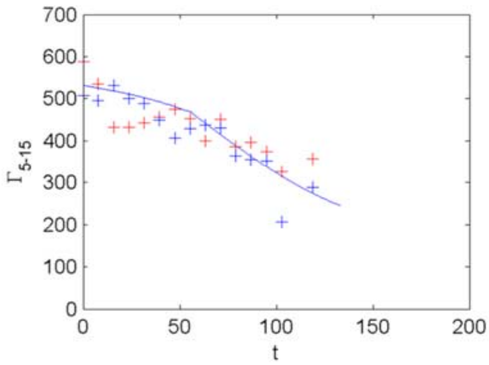
The physical transport models used in the DVM

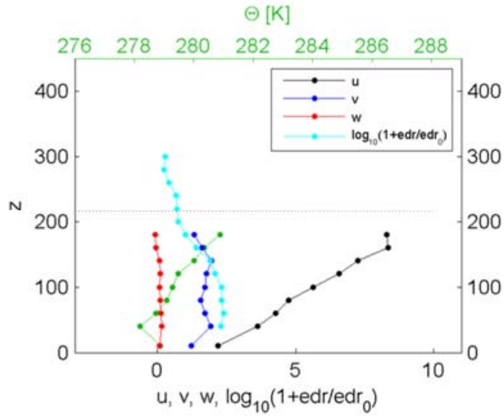
- Self-induced velocity: Biot-Savart law also using image particles (for NGE modeling)
- Wind convection: by both the axial and lateral components evaluated at the altitude of WV evolution
- Wind shear: the shear of the wind profile is computed and the tilting effect is modeled
- Stratification effects: the stratification level, N*, is computed from the temperature profile and the thermally induced rebound is modeled



The physical models used in the DVM

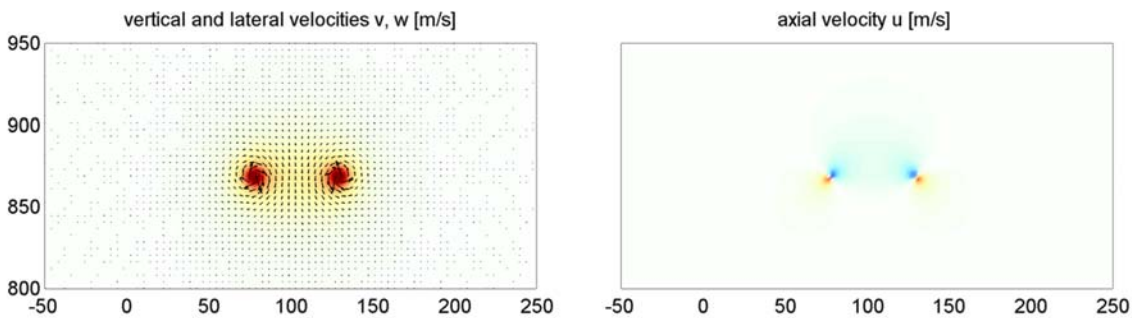
- Circulation decay
  - Two phase decay: slow decay phase followed by a rapid decay phase
  - Accumulated "time-to-demise" approach: starting time of the fast decay phase t_d(EDR, N*)
  - The decay rate of both phases also depends on EDR and N*
  - Possibility to also use a TKE-based decay model or the APA decay model using EDR



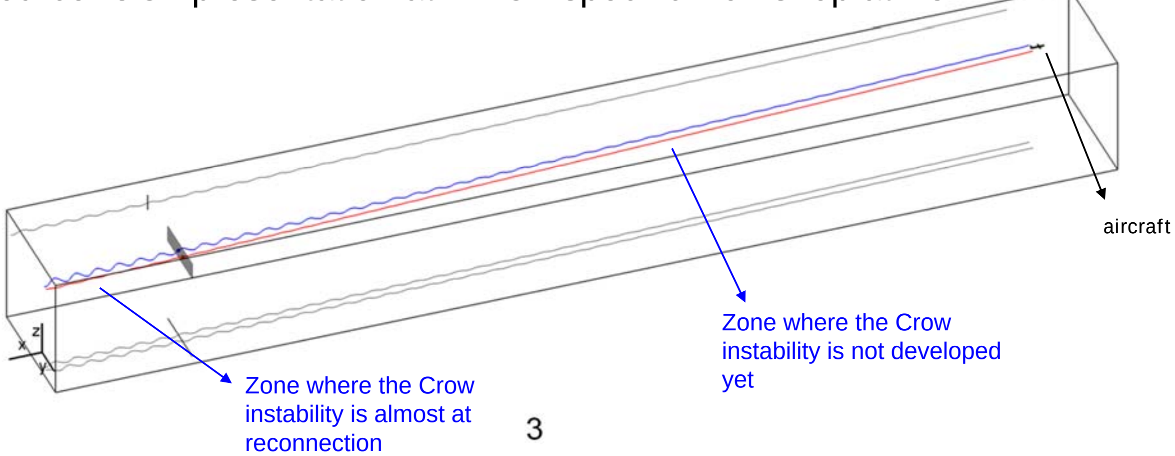


- Crow instability model
  - Model of the space developing Crow instability amplitude for predicting the WV deformation
  - Based on the time-to-demise and thus the atmosphere turbulence level

Velocity field evaluation module: case using WAKE4D results with Crow instability effects (case with significant atm turbulence)



Source: UCL presentation at WN3E specific workshop at TUB



Application of DVM to relative comparison of a/c OGE

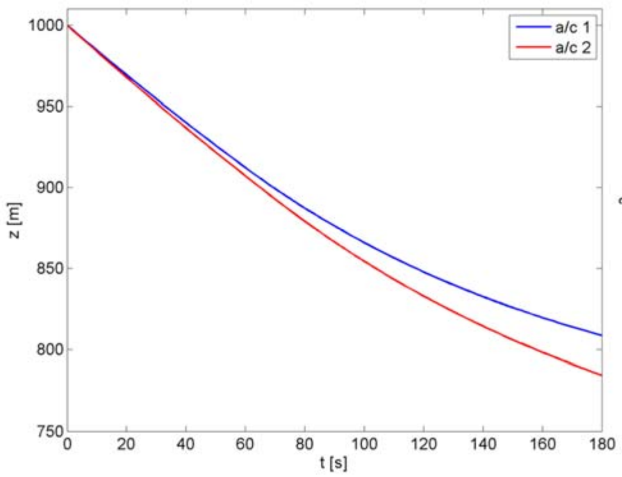
Variation of weight, wingspan for constant met. conditions : no wind, turbulence ($\epsilon = 7\,10^{-4}\,m^2/s^3$) and no stratification

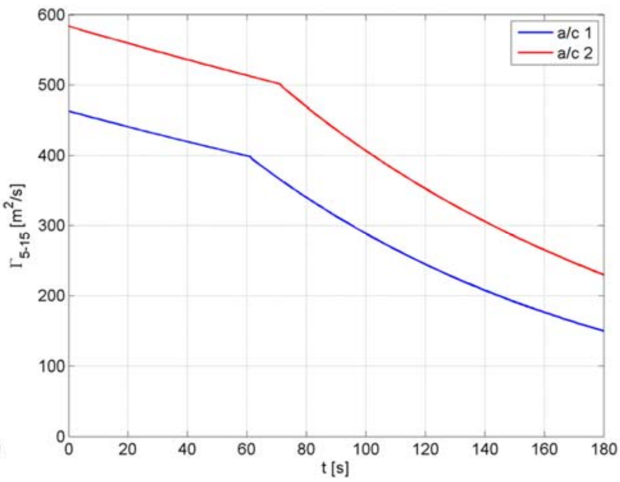
Example where same loading factor was assumed for both aircraft:

$$s = \frac{b_0}{b} = \frac{\pi}{4} = 0.785$$

$$\epsilon_1^* = \frac{(\epsilon\, b_{0,1})^{1/3}}{V_{0,1}} = 0.209$$

$$\epsilon_2^* = \frac{(\epsilon\, b_{0,2})^{1/3}}{V_{0,2}} = 0.214$$

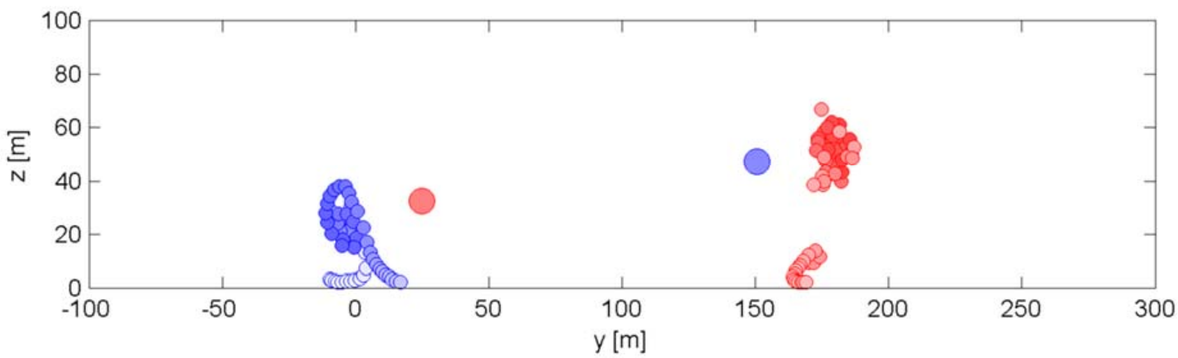




Source: UCL presentation at WN3E 1st major workshop

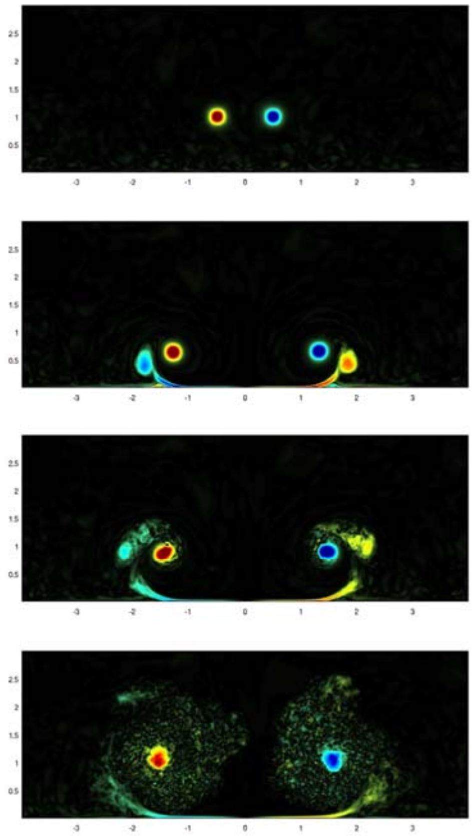
DVM In Ground Effect (IGE) modeling

- Generate new « secondary » vortex particles close to the ground, to model the ground-generated boundary layer due to no-slip
- Those particles also dynamically separate from the ground
- They interact with the primary vortices and induce the rebound
- There is a redistribution of the secondary particles when the primary vortex has bounced
- An additional IGE decay model, based on a “Particle strength exchange”, enhances the decay of the vortices IGE after rebound.

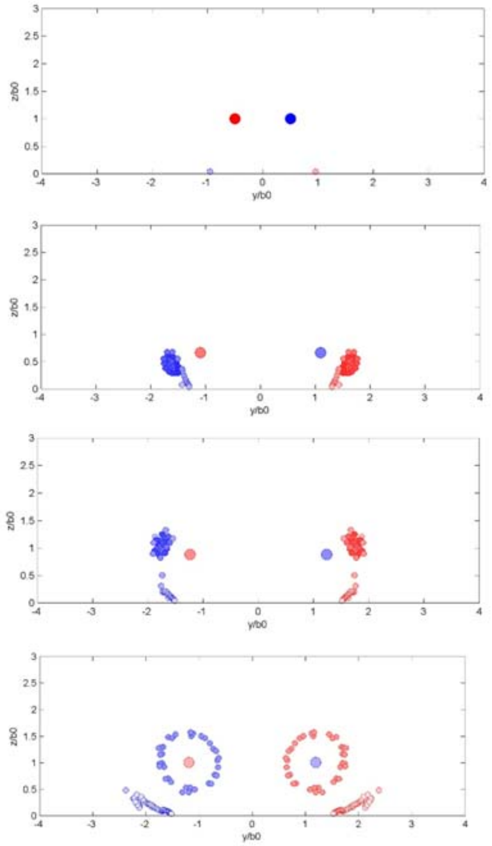


Comparison between LES and DVM results: case IGE with headwind (from FAR-Wake)

Mean axial vorticity field as computed by a LES



Particles used in the DVM to model WV IGE

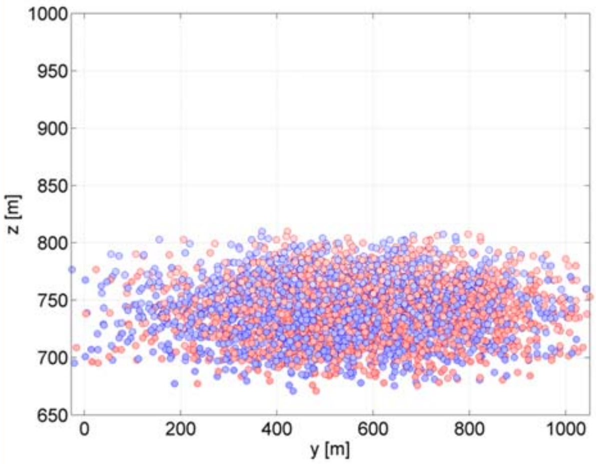


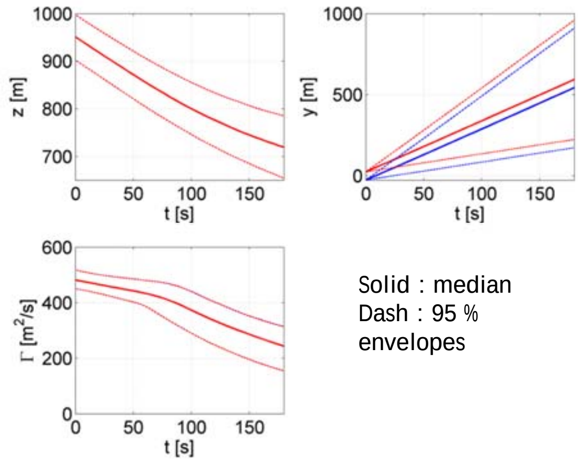
The Probabilistic wake Vortex Model (PVM)

- Probabilistic modeling and assessment of wake vortices is operationally required.
- The PVM is an upper software layer, based on a Monte-Carlo approach, using the DVM as a subtool.
- For each PVM run, many DVM runs are performed, with random variations on the impact parameters (each one following its own distribution):
  - met conditions (natural variations and uncertainties)
  - a/c characteristics (uncertainties)
  - some coefficients of the physical models (calibration uncertainties)

PVM outputs

- One then obtains, in a computational gate at each time:
  - a set of vortex positions,
  - their associated circulations.
- Thus: the output distribution is not just a simple image of the input distribution
- A statistical analysis on the result sample can be performed to obtain:
  - PDF, mean, median, variance, percentiles, …

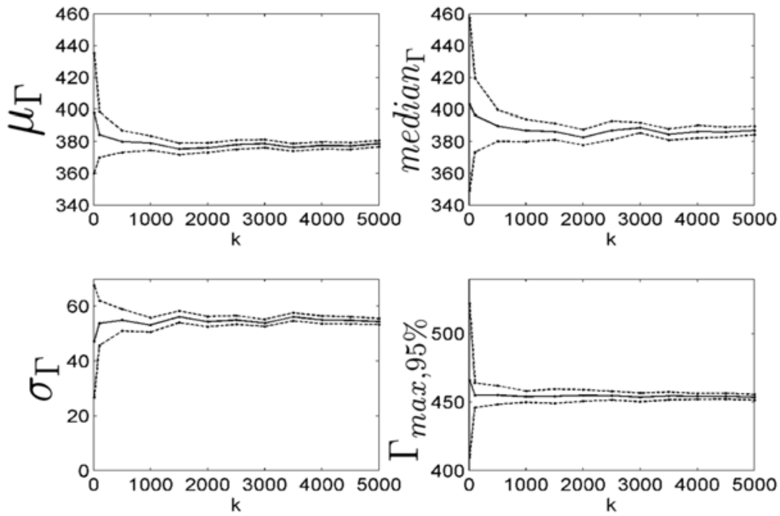




Bootstrap resampling technique

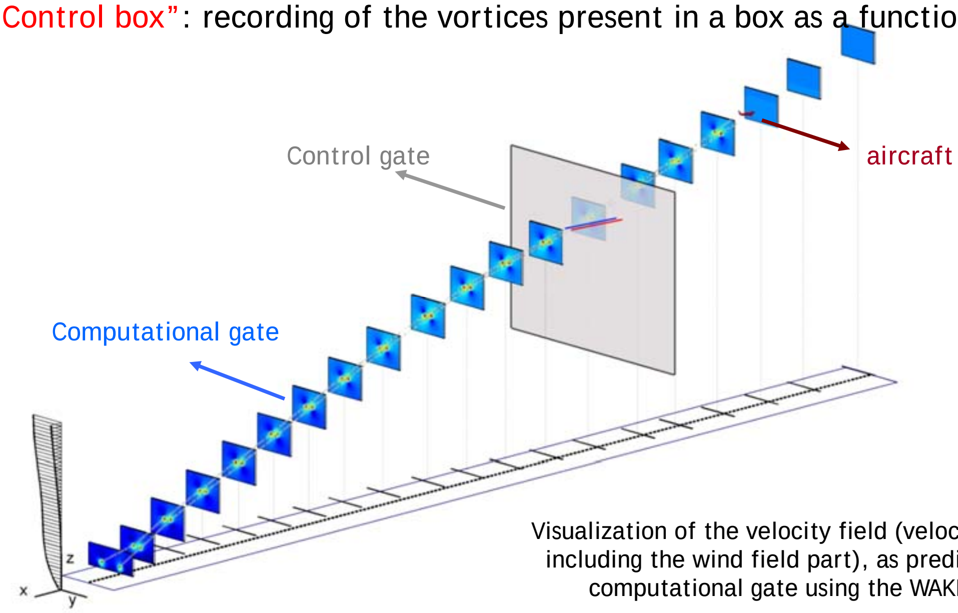
- Aim : obtain conservative statistical results while limiting the number of DVM runs
- The resampling technique provides an estimate of the variance of the statistics of the Monte-Carlo results
- The “statistics on the statistics” converge fast

  => PVM confidence envelopes are accurate using a moderate number of DVM runs.
- Thus: the PVM approach is computationally efficient, also for real-time systems.



WAKE4D-DVM and WAKE4D-PVM

- The platform can be run deterministically (using the DVM in each gate) or probabilistically (using the PVM in each gate).
- From the 3-D “gate by gate” DVM (resp. PVM) computations, one can rebuild the 3-D wake (resp. envelope of the wake).
- “Control gate”: interpolation in a fixed plane (similar to a LIDAR scanning plane).
- “Control box”: recording of the vortices present in a box as a function of time.

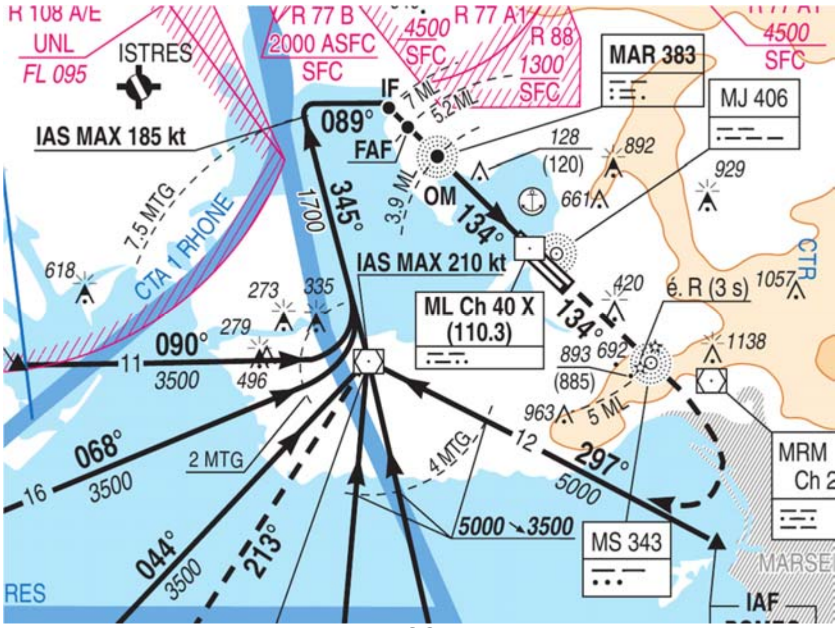


Visualization of the velocity field (velocity norm, also including the wind field part), as predicted in each computational gate using the WAKE4D-DVM

Example of use of WAKE4D : approach to Marseille-Provence

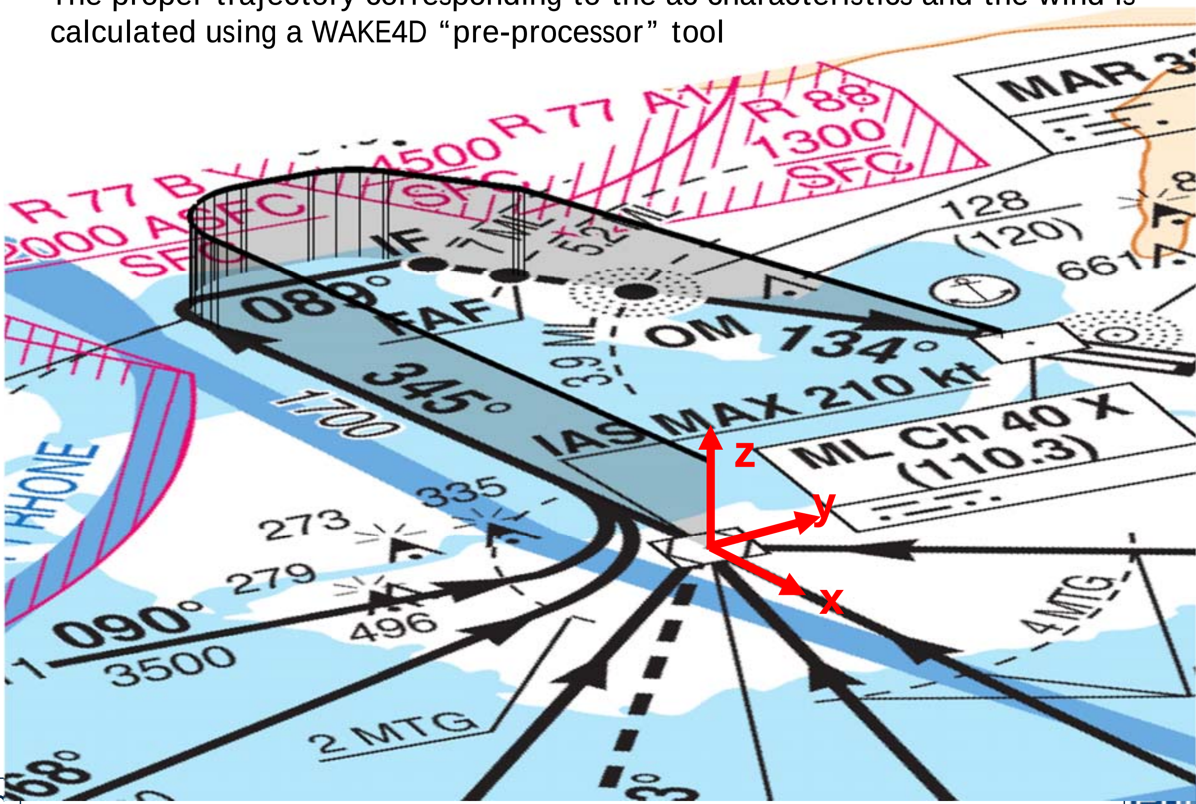
- Aircraft : B747- 400
  - MLW : 285,000 kg
  - b=64.4 m
  - s= 0.75 [-]

- Met. conditions :
  - Southern wind profile (log profile)
  - Turbulence (EDR=10^{-4} m^2/s^3 OGE, then log law)
  - No stratification



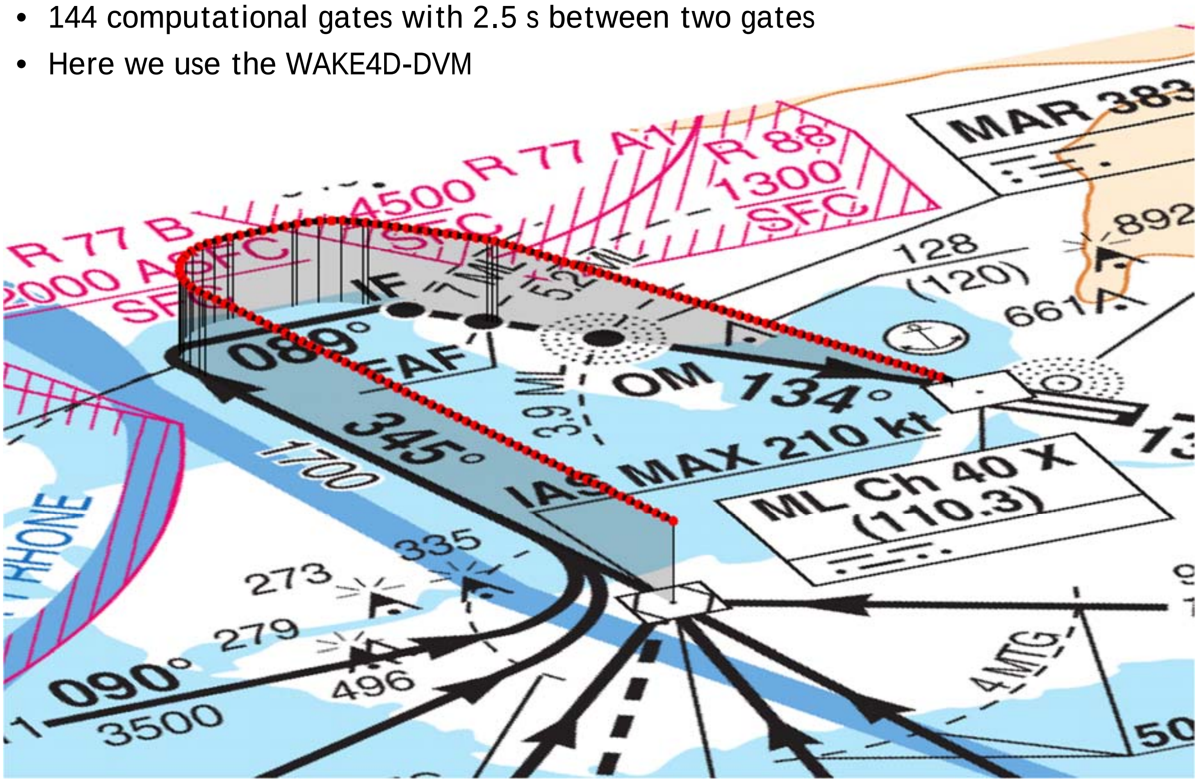
Aircraft trajectory

- The proper trajectory corresponding to the ac characteristics and the wind is calculated using a WAKE4D “pre-processor” tool



Computational gate locations

- 144 computational gates with 2.5 s between two gates
- Here we use the WAKE4D-DVM



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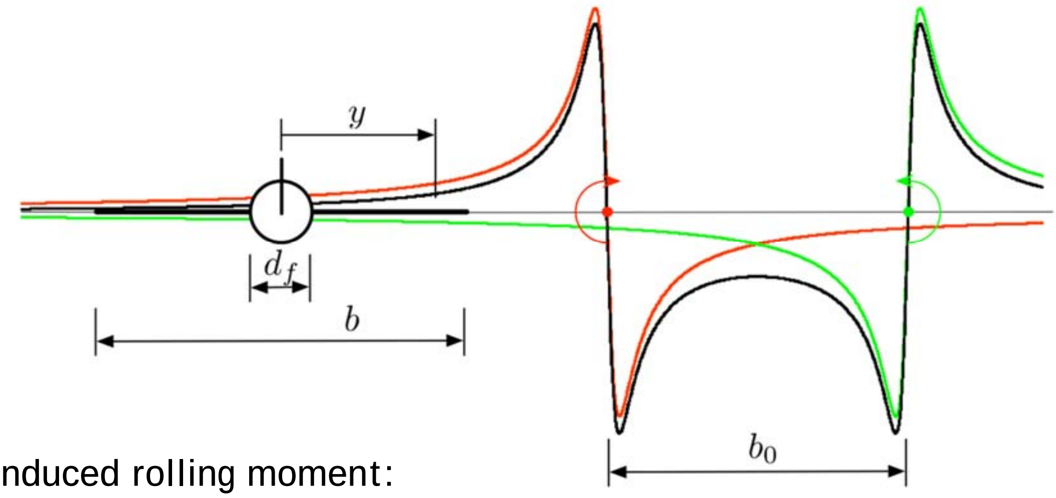
Rolling moment induced by a 2VS on a follower aircraft



- Induced rolling moment:

$$M = \frac{1}{2}\rho U_\infty^2 \int_{-b/2}^{b/2} \frac{w_v(y)}{U_\infty}\, C_{l,\alpha}(y)\, c(y)\, y\, dy$$

$$C_M = \frac{M}{\frac{1}{2}\rho U_\infty^2\, S\, b} = \int_{-b/2}^{b/2} \frac{w_v(y)}{U_\infty}\, C_{l,\alpha}(y)\, \frac{c(y)}{\overline{c}}\, \frac{y}{b}\, \frac{dy}{b} \qquad S = b\,\overline{c}$$

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Further assumptions

- Wing with uniform lift slope: $C_{l,\alpha}(y) = C_{l,\alpha}$
- Wing with linear taper: $\frac{c(y)}{\overline{c}} = (1+\beta) - 2\,\beta\,\frac{|y|}{b/2}$
- Remove the non-contribution of the fuselage of diameter d_f
- One then obtains:

$$\begin{aligned} C_M \;&=\; C_{l,\alpha}\left[\int_{-b/2}^{-d_f/2} \frac{w_v(y)}{U_\infty}\left((1+\beta)+4\,\beta\,\frac{y}{b}\right)\frac{y}{b}\,\frac{dy}{b}\right. \\ &\qquad\left. + \int_{d_f/2}^{b/2} \frac{w_v(y)}{U_\infty}\left((1+\beta)-4\,\beta\,\frac{y}{b}\right)\frac{y}{b}\,\frac{dy}{b}\right] \end{aligned}$$

Model used so far in simplified analysis

- For simplicity, we here assume that each longitudinally averaged vortex is well represented by a low order algebraic (LOA = B-H) circulation profile:

$$\Gamma(r,t) = \Gamma_0(t)\,\frac{r^2}{(r^2 + r_c^2(t))}$$

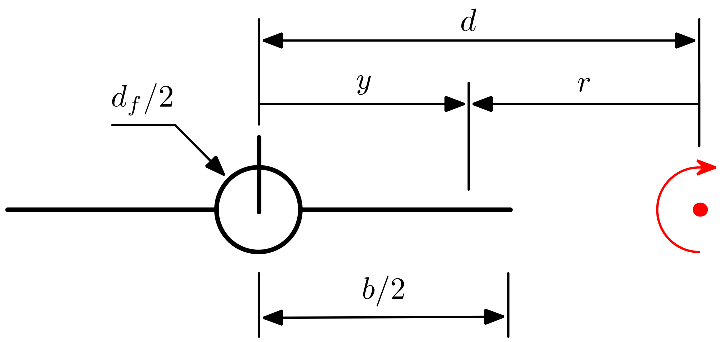
- The induced azimuthal velocity due to one vortex is then:

$$u_\theta(r,t) = \frac{\Gamma(r,t)}{2\pi\, r} = \frac{\Gamma_0(t)}{2\pi}\,\frac{r}{(r^2 + r_c^2(t))} \qquad\qquad u_\theta(r_c(t)) = \frac{\Gamma_0(t)}{4\pi\, r_c(t)}$$

- The other models could be investigated as well (not done yet)

Case with wing fully to the left of the left (port) vortex center: contribution of that vortex

- Case $d \geq \frac{b}{2}$:



$$-\frac{b}{2} \leq y \leq \frac{b}{2} \;:\quad y + r = d \quad\Rightarrow\quad r = d - y$$

- One then obtains:

$$C_M = \frac{C_{l,\alpha}}{2\pi}\,\frac{\Gamma_0}{U_\infty\, b}\left[\int_{-b/2}^{-d_f/2} \frac{(d-y)}{((d-y)^2 + r_c^2)}\left((1+\beta) + 4\,\beta\,\frac{y}{b}\right)\frac{y}{b}\,dy + \int_{d_f/2}^{b/2} \frac{(d-y)}{((d-y)^2 + r_c^2)}\left((1+\beta) - 4\,\beta\,\frac{y}{b}\right)\frac{y}{b}\,dy\right]$$

- All other cases can be analyzed as well: this was done

B747-400 leader

$$\begin{aligned} b &= 64.44 \text{ m} \\ S &= 541.2 \text{ m}^2 \\ MTOW &= 396 \text{ tons} \end{aligned}$$

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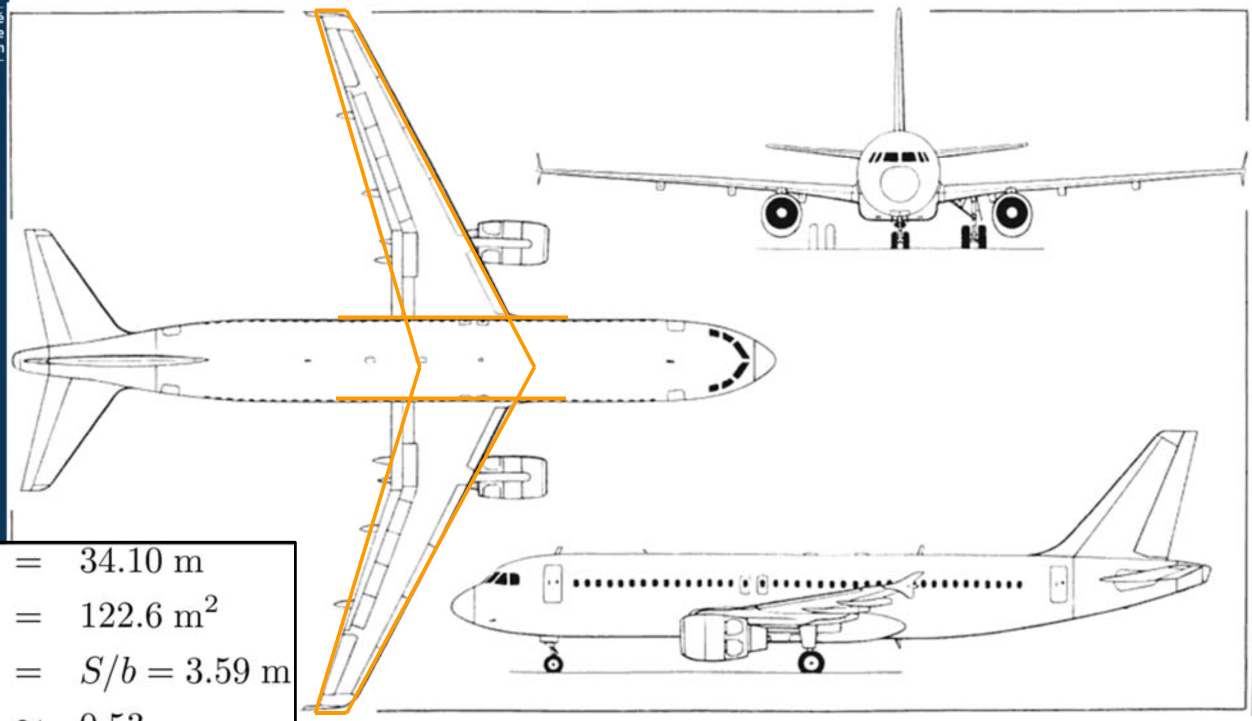
A380-800 leader

$$\begin{aligned} b &= 79.8 \text{ m} \\ S &= 845 \text{ m}^2 \\ MTOW &= 560 \text{ tons} \end{aligned}$$

Exemple with A320 follower (medium)



$$
\begin{aligned}
b &= 34.10 \text{ m} \\
S &= 122.6 \text{ m}^2 \\
\bar{c} &= S/b = 3.59 \text{ m} \\
\beta &\approx 0.53 \\
d_f &= 3.95 \text{ m}
\end{aligned}
$$

Relative comparisons

- Compute the following dimensionless quantities:
  - a/c encountering the wake of a/c 1 (B747-400):

$$\frac{U_\infty b}{\Gamma_{0,1}} \frac{2\pi}{C_{l,\alpha}} C_{M,1} = G\left(\frac{d}{b}; \frac{d_f}{b}, \frac{r_{c,1}}{b}, \frac{b_{0,1}}{b}, \beta\right) = G_1\left(\frac{d}{b}\right)$$

  - a/c encountering the wake of a/c 2 (A380-800):

$$\frac{U_\infty b}{\Gamma_{0,2}} \frac{2\pi}{C_{l,\alpha}} C_{M,2} = G\left(\frac{d}{b}; \frac{d_f}{b}, \frac{r_{c,2}}{b}, \frac{b_{0,2}}{b}, \beta\right) = G_2\left(\frac{d}{b}\right)$$

- Assume a ratio for the circulations:

$$\frac{\Gamma_{0,2}}{\Gamma_{0,1}} = \gamma > 1$$

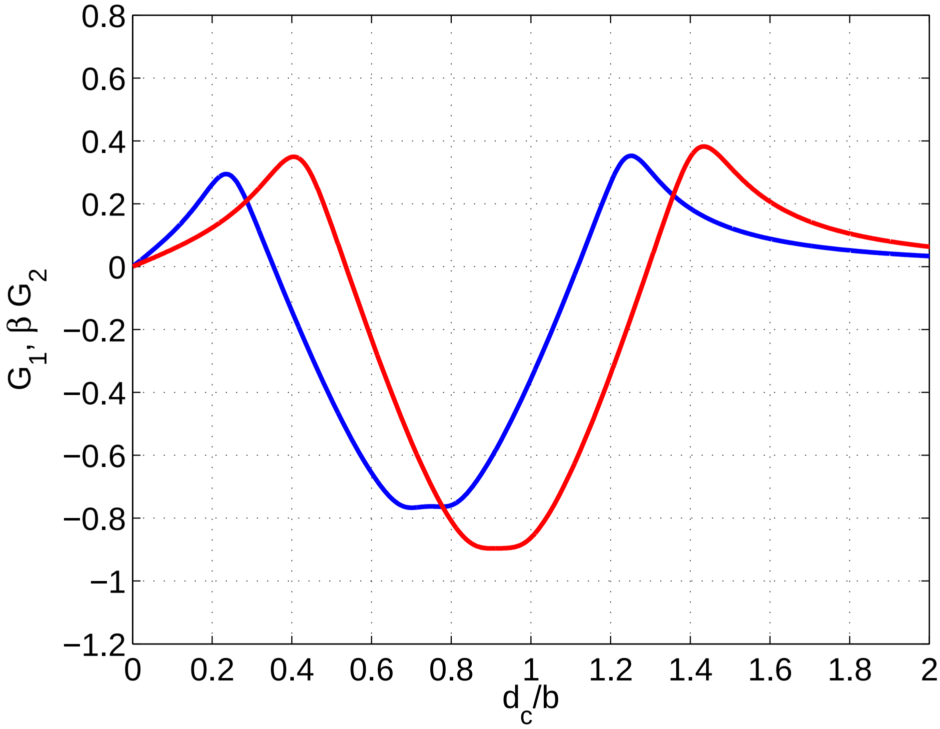
- Hence:

$$\frac{U_\infty b}{\Gamma_{0,1}} \frac{2\pi}{C_{l,\alpha}} C_{M,2} = \gamma\, G_2\left(\frac{d}{b}\right)$$

Plots to be compared!

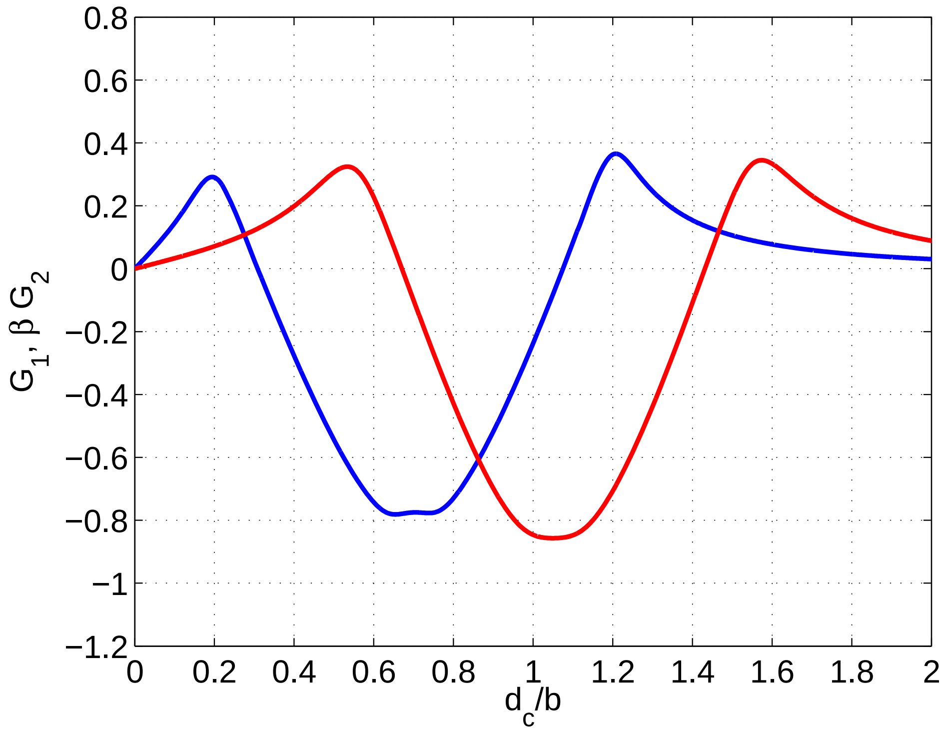
A320 follower, wrt 2VS center: $d_c = d + b_0/2$

$$\frac{\Gamma_{0,2}}{\Gamma_{0,1}} = 1.25\ , \quad \frac{b_{0_1}}{b_1} = \frac{b_{0_2}}{b_2} = 0.785\ , \quad \frac{r_{c_1}}{b_{0,1}} = \frac{r_{c_2}}{b_{0,2}} = 0.04$$



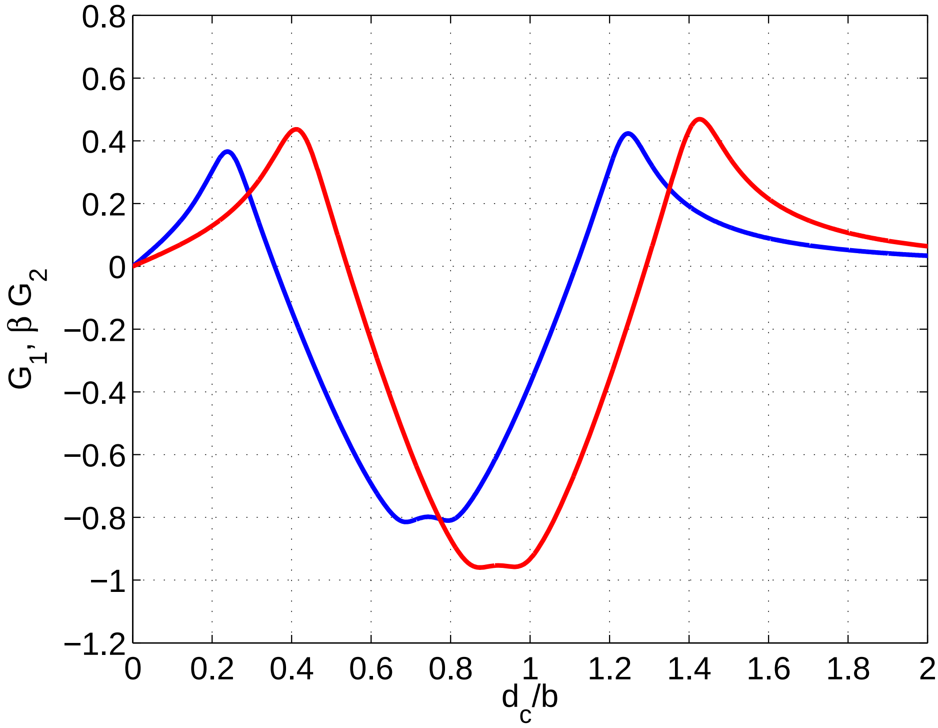
A320 follower, wrt 2VS center

$$\frac{\Gamma_{0,2}}{\Gamma_{0,1}} = 1.25\ , \quad \frac{b_{0_1}}{b_1} = 0.74\ , \quad \frac{b_{0_2}}{b_2} = 0.90\ , \quad \frac{r_{c_1}}{b_{0,1}} = \frac{r_{c_2}}{b_{0,2}} = 0.04$$



A320 follower, wrt 2VS center

$$\frac{\Gamma_{0,2}}{\Gamma_{0,1}} = 1.25 \ , \quad \frac{b_{0_1}}{b_1} = \frac{b_{0_2}}{b_2} = 0.785 \ , \quad \frac{r_{c_1}}{b_{0,1}} = \frac{r_{c_2}}{b_{0,2}} = 0.03$$



A320 follower, wrt 2VS center

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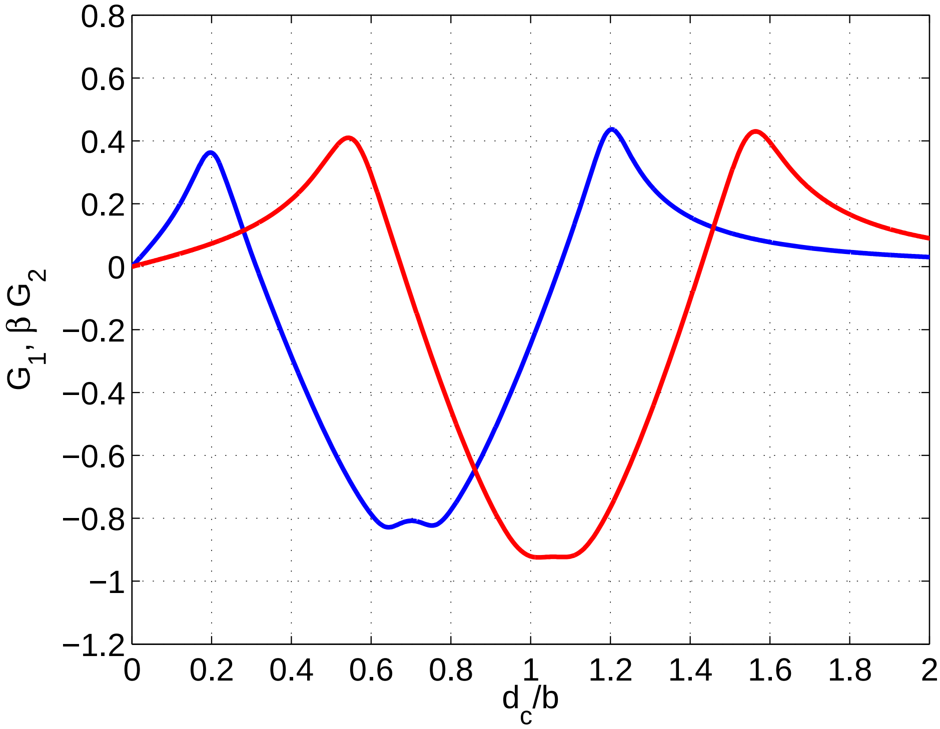

Conclusion of simplified static analysis

- Static analysis of rolling moment induced by 2VS (i.e., before Crow instabilities)
- Leaders: A380-800 and B747-400, with same r_c/b_0
- Followers: A320 and A300B2
- Even when assuming 25% more circulation for the A380-800 (which is really an upper bound):
  - the rolling moment when the outer part of the wing is within one vortex core (i.e., case d/b=0.5) is « much the same ». This is even more so when using more realistic values of b_0/b.
  - The rolling moment when the a/c center is inside one vortex core (i.e., case d/b=0) is still higher, yet by less than 25%. This is even more so when using more realistic values of b_0/b.
- Hence: circulation is not the whole story: there is also r_c ; and b_0/b.