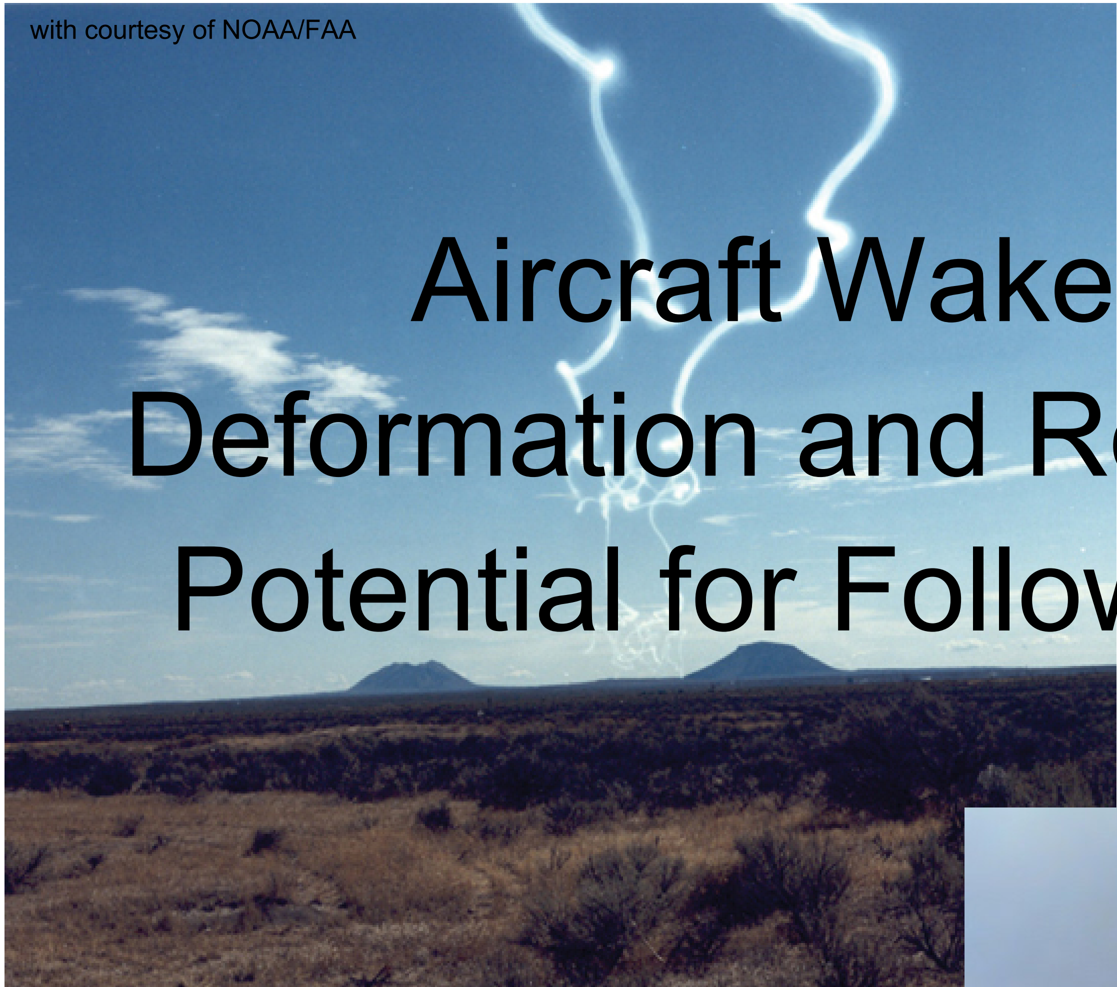

with courtesy of NOAA/FAA

Aircraft Wake Vortex Deformation and Resulting Risk Potential for Following Aircraft

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Institute of Atmospheric Physics
DLR Oberpfaffenhofen, Germany

with courtesy of Airliners.net

Goals & Outline

1) State-of-the-art of current P2P (Probabilistic 2-Phase Decay Model) model as central part of WSVBS
2) Motivation for new LES
    1) new 3-D vortex position algorithm
    2) analysis of circulation evolution (along vortex path ↔ along flight direction)
3) Preliminary inclusion of 3-D effects (P2P → P3P)
4) Risk estimation for following aircraft (strip theory)

D2P / P2P

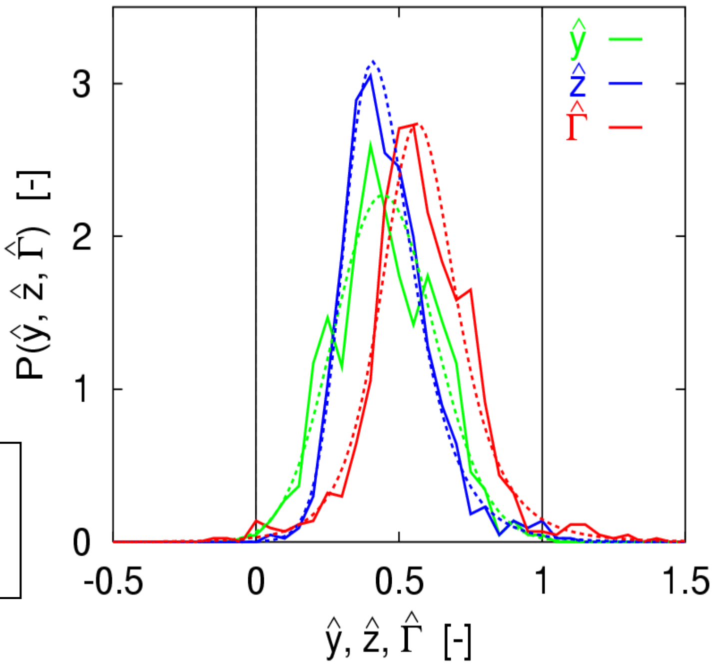
- Modeling of vortex evolution (z^*, y^*, Γ^*_{5-15}) in given atmospheric conditions (ε^*, N^*)
- Deterministic:

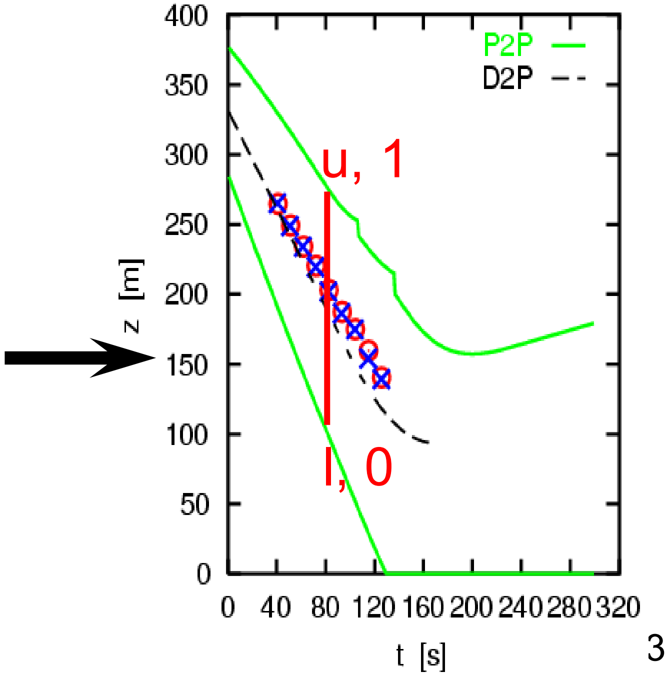
$$\Gamma^*_{5-15}(t^*) = A - \exp\left(\frac{-R^2}{\nu_1^*\left(t^* - T_1^*\right)}\right) - \exp\left(\frac{-R^2}{\nu_2^*\left(t^* - T_2^*\right)}\right)$$

$$\nu_1^*, T_1^*, \nu_2^*, T_2^* = f(\varepsilon, N)$$

- Probabilistic:

unbounded Johnson distribution





Wake Vortex Prediction and Monitoring System (WSVBS)

WSVBS

meteo measurements
SODAR/RASS USA
3 gates, 0.3 - 1 NM

numerical weather prediction
NOWVIV
10 gates, 2 - 11 NM

wake-vortex prediction
P2P
envelopes for y(t), z(t), Γ(t) in 13 gates
for 8 a/c parameter classes (heavy)

glide path adherence statistics
FLIP
standard deviations in 13 gates

safety area prediction
SHAPe
ellipses in 13 gates for HH, HM

wake-vortex monitoring
LIDAR
3 planes, 0.3 - 1 NM

temporal a/c separations
for 6 a/c-class combinations

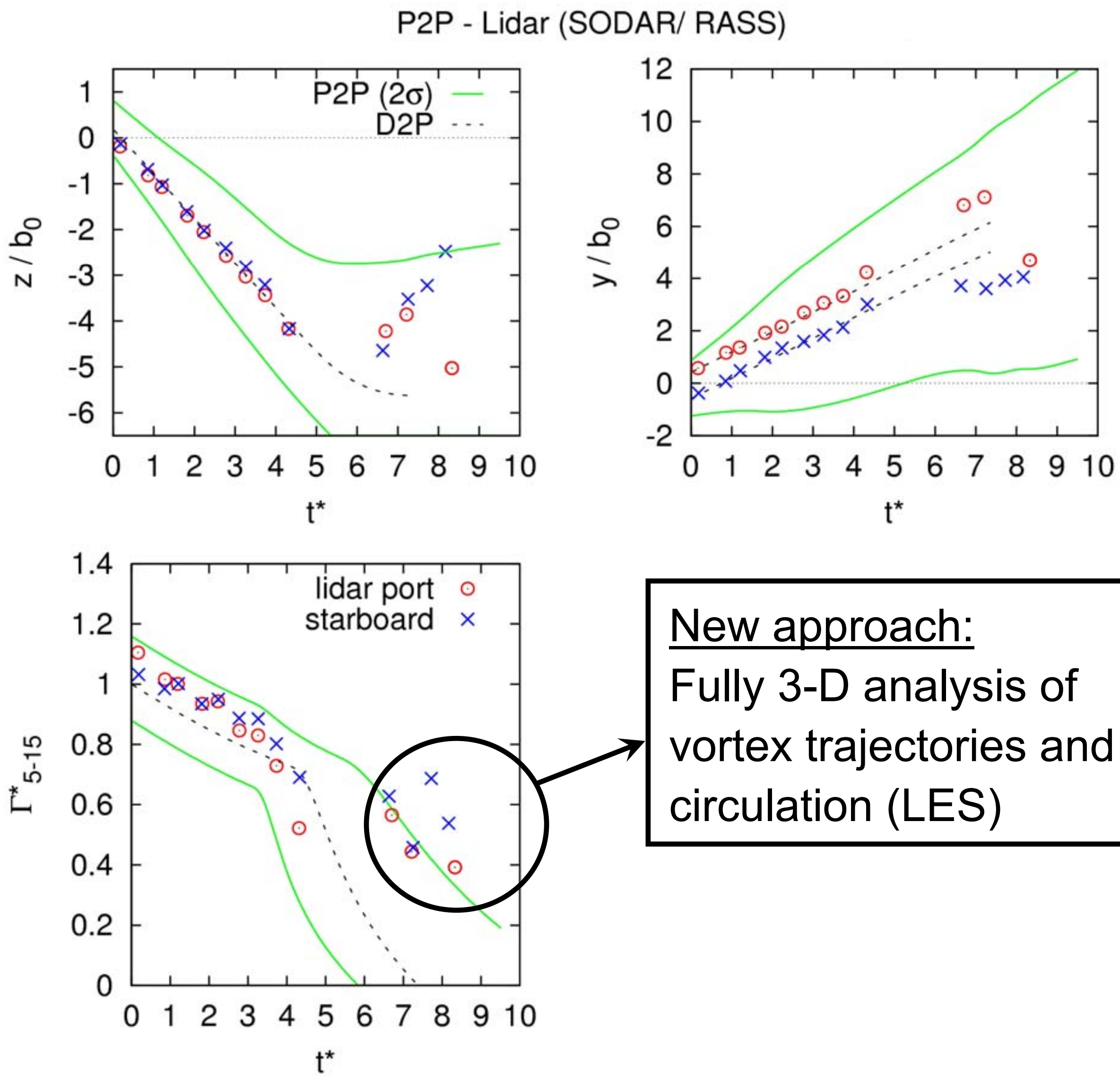
conflict detection
issue warning / adapt predictions

procedures
AMAN
STG, MSR, MSL, ICAO

(Holzäpfel et al)

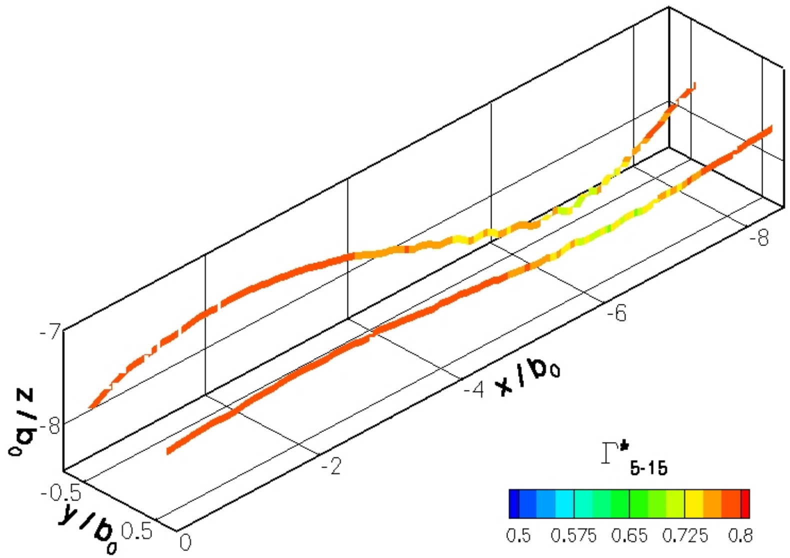
Motivation for 3-D consideration

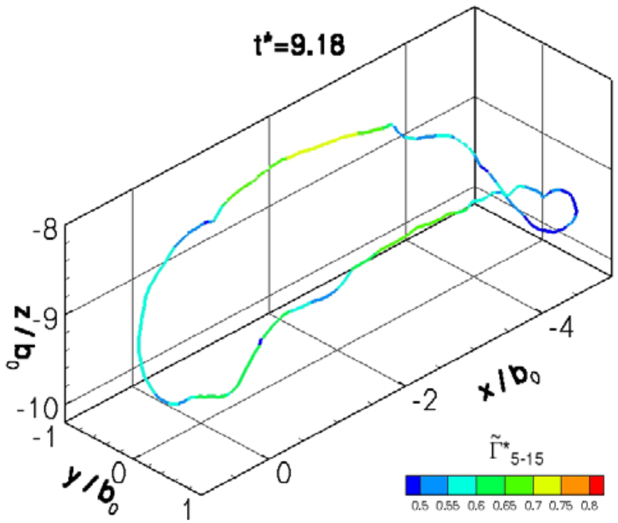
- Lidar measurements from AWIATOR (FT 2-04, 2003)
- P2P prediction for z, y und Γ based on
  - former LES & Lidar data
  - meteorological measurement data



New approach:
Fully 3-D analysis of vortex trajectories and circulation (LES)

Vortex Instabilities & Numerics



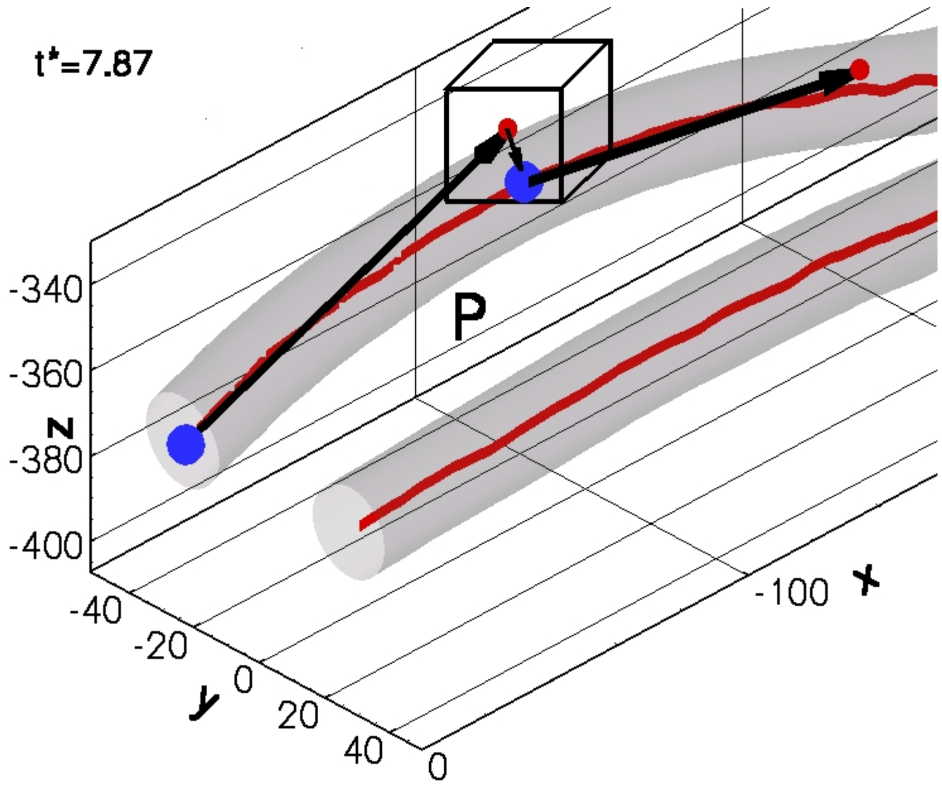


LES using LESTUF:

- Temperature stratification with normalized Brunt-Väisälä-Frequency N* [0, 0.35, 1]
- Normalized turbulent dissipation rate ε* [0.01, 0.05, 0.23]
- Normalized quantities for time, length, velocities and circulation

$$t_0 = \frac{b_0}{w_0} \approx 30s\,|_{A340}$$

New Vortex Path Detection (predictor-corrector algorithm)



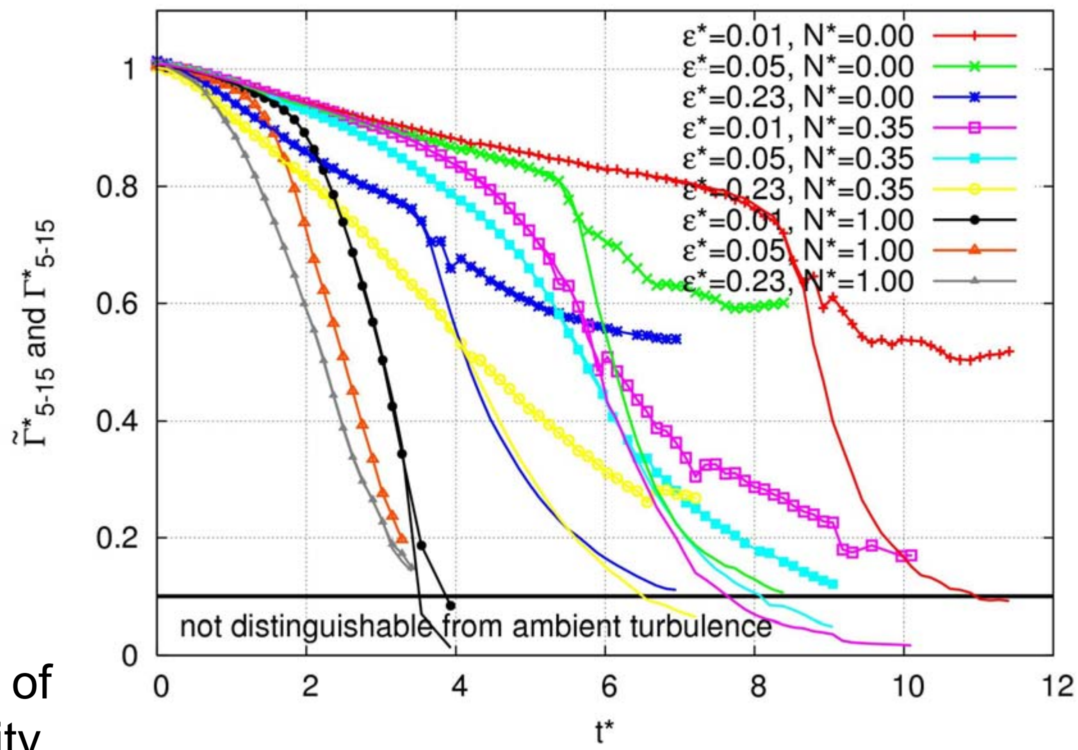
Algorithm is completely independent of flight direction!

1) Chooses starting point with P-minimum
2) Determines vorticity vector
3) Creates cube around vorticity direction end point
4) Searches pressure minimum for next vortex track point

$$\vec{\bar{x}} = \frac{\iiint \vec{x}\ P_{(\vec{x})} dV}{\iiint P_{(\vec{x})} dV}$$

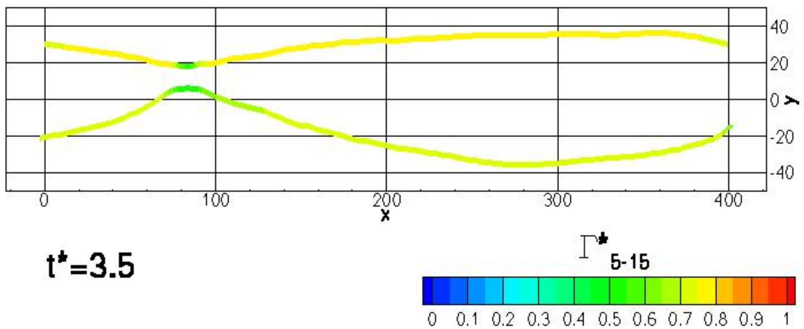
5) Determines next vorticity vector

Vortex Circulation Evolution



ε*>>0:

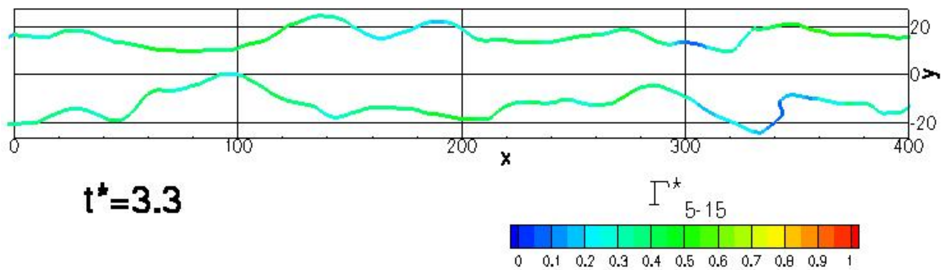
strong forcing of Crow instability



t*=3.5

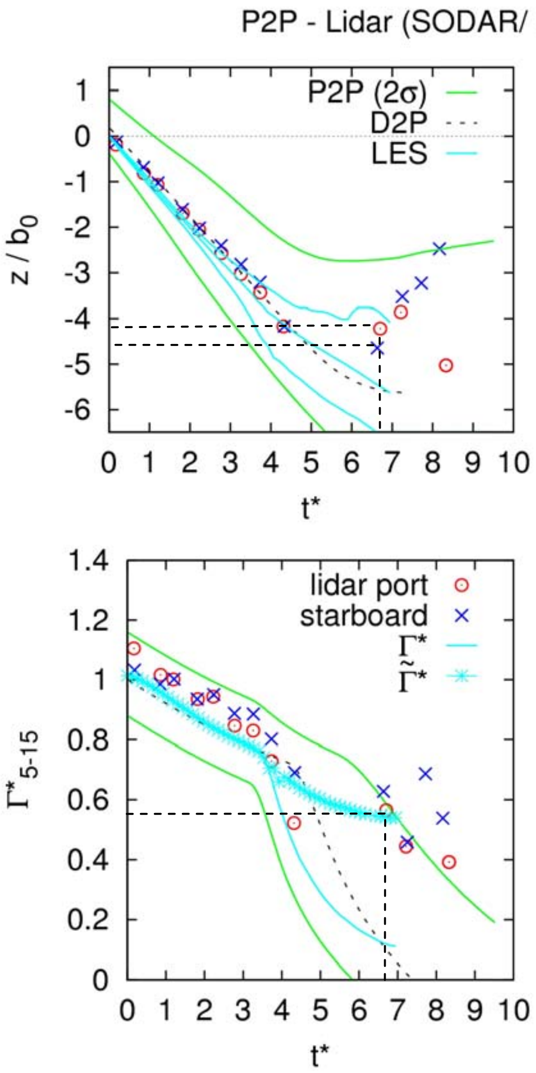
N*>>0:

$$\frac{D\omega_{BV}}{Dt} = \frac{1}{\rho^2}\nabla\rho \times \nabla p$$



t*=3.3

Validation AWIATOR

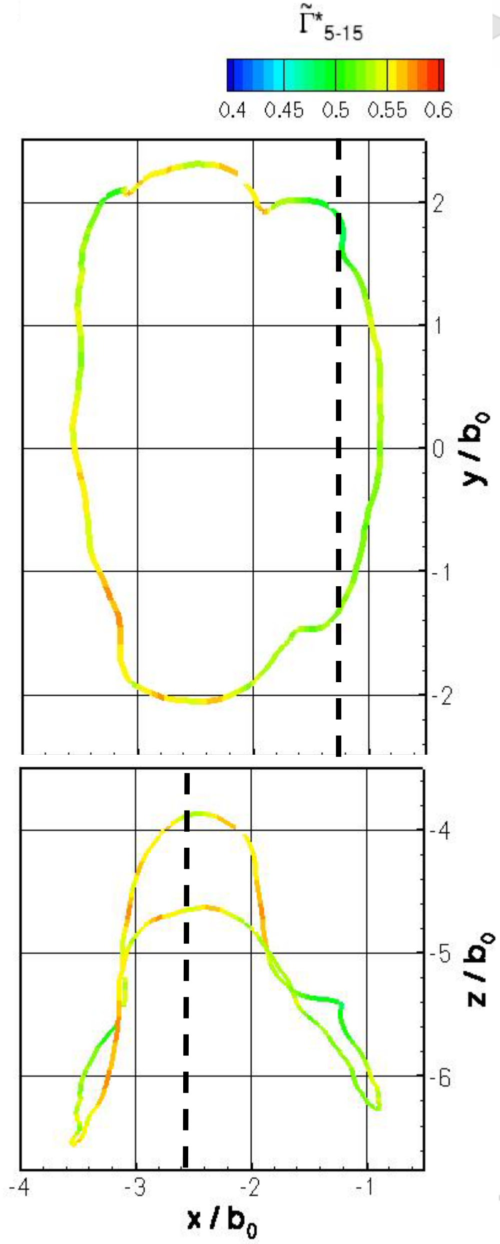


LES:

N*=0

ε*=0.23

t*=6.7



P3P…

… shall improve vortex position and circulation prediction at late times

… is based on core elements of P2P and shall therefore keep:

- the diffusion term
- the atmospheric parameters ε* and N* as input (L_t?)
- $\Gamma_{5\text{-}15}$ as a measure for risk potential

… is completed by a modeling term which accounts for the formation of vortex rings (e.g. a tanh-term)

… shall use the uncertainty of parameters (ε*, N*, T*) for probabilistic envelopes

$$\Gamma^*_{5-15}(t^*) = A - \exp\left(\frac{-R^2}{\nu_1^*\left(t^* - T_1^*\right)}\right) - \exp\left(\frac{-R^2}{\nu_2^*\left(t^* - T_2^*\right)}\right)$$

$$\nu_1^*, T_1^*, \nu_2^*, T_2^* = f(\varepsilon, N) \longrightarrow \nu_1^*, T_1^*, ... = f(\varepsilon, N, L_t)$$

Resulting Risk Potential

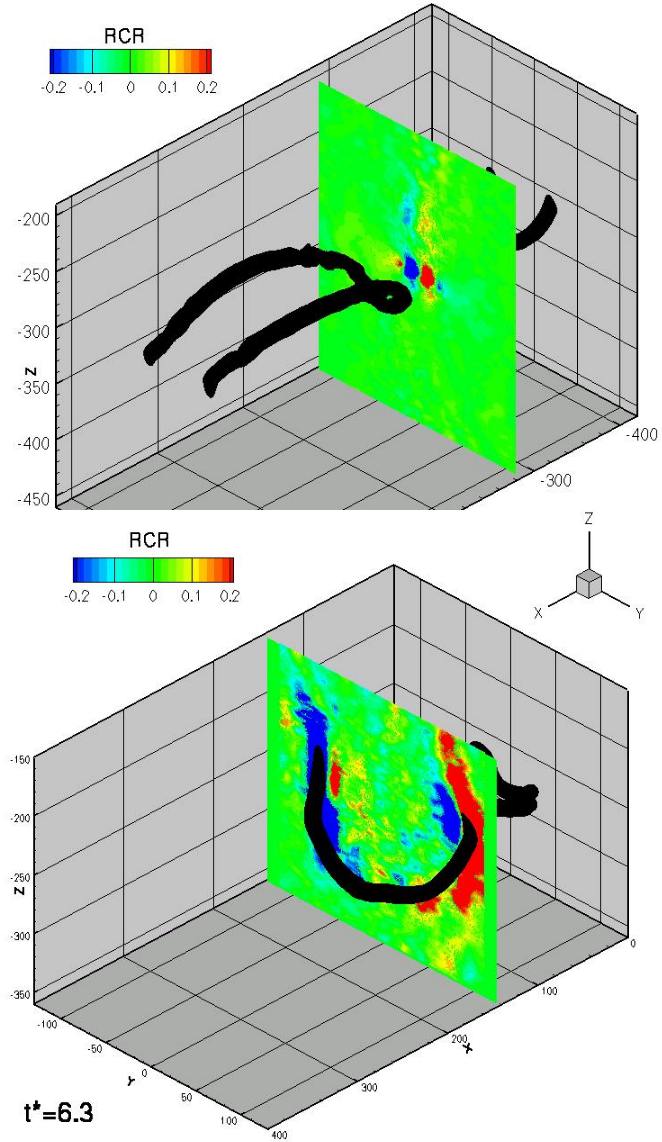
- **Strip theory:**

  Calculation of rolling coefficient C_r

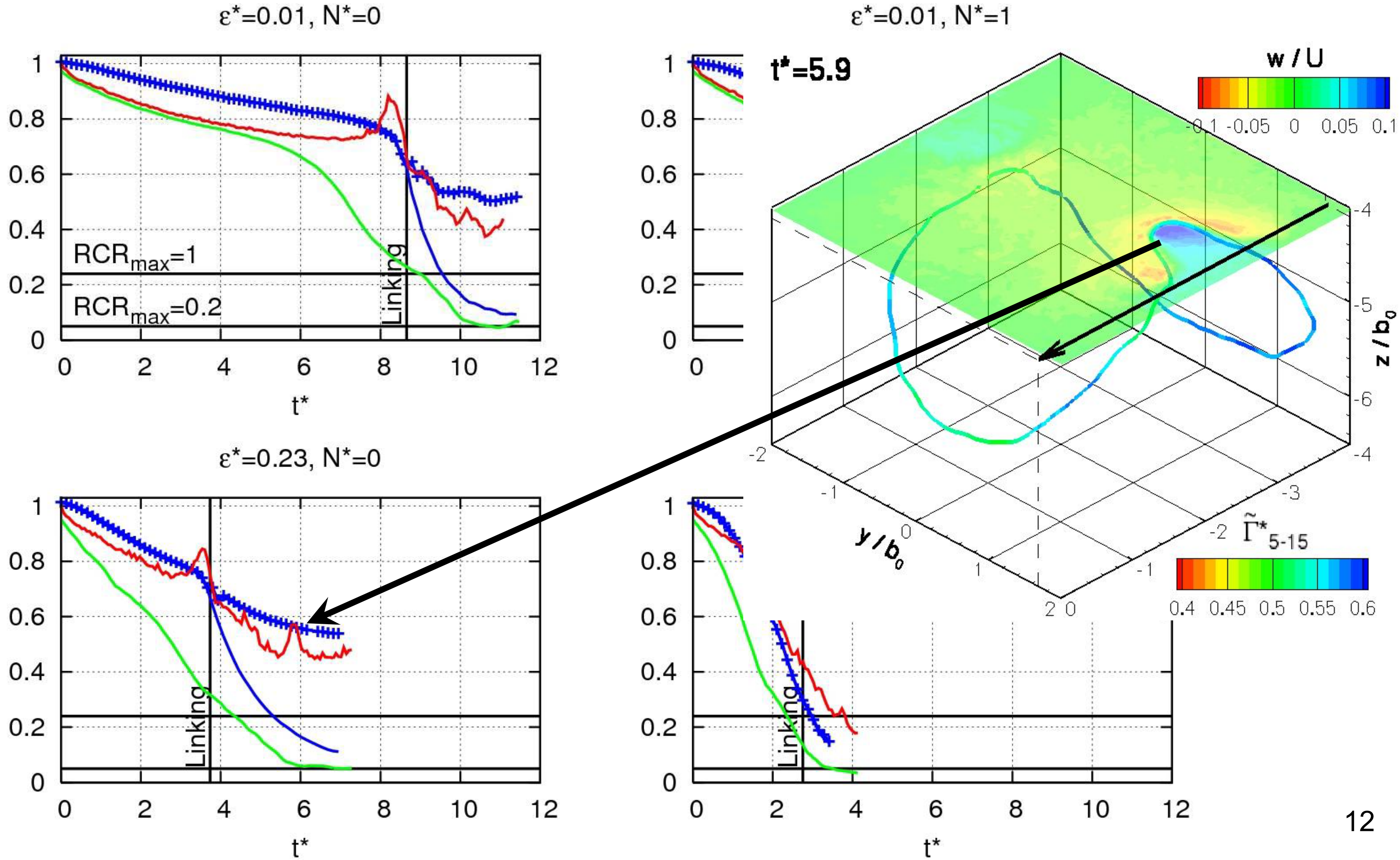
  α_{eff} depends on velocity field

$$C_{r,WV} = \frac{M}{qS\,B/2} = \frac{4\pi}{SB}\int_{-B/2}^{B/2} c(y)\alpha_{eff}(y)\,y\,dy$$

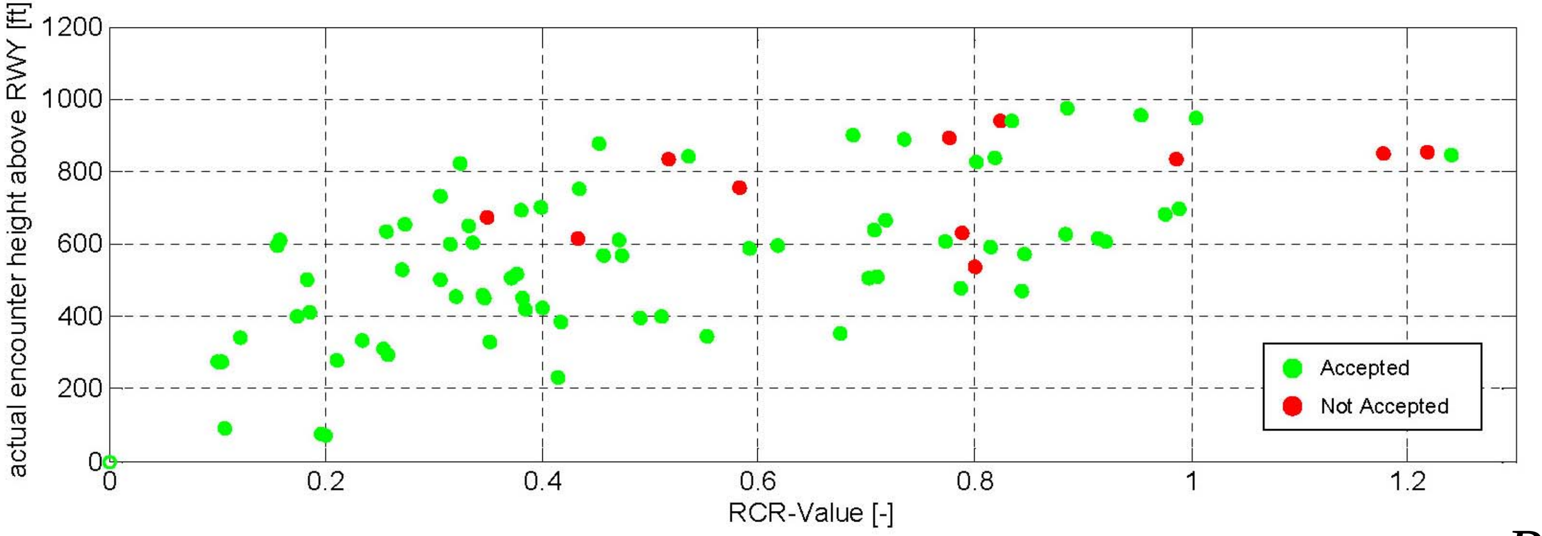
- A340-300 followed by B737-300 (H-M)
- $C_{r,max}|_{B737}=0.06$
- Roll Control Ratio: $$RCR = \frac{C_{r,WV}}{C_{r,a/c}}$$



Rolling Coefficient C_r

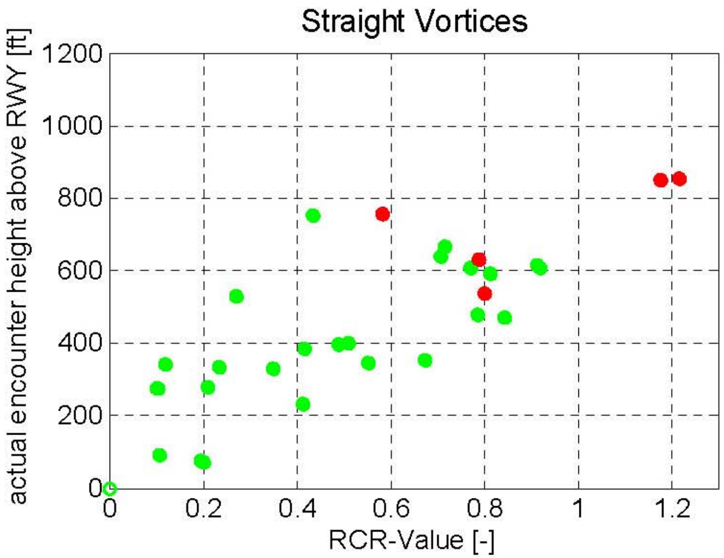


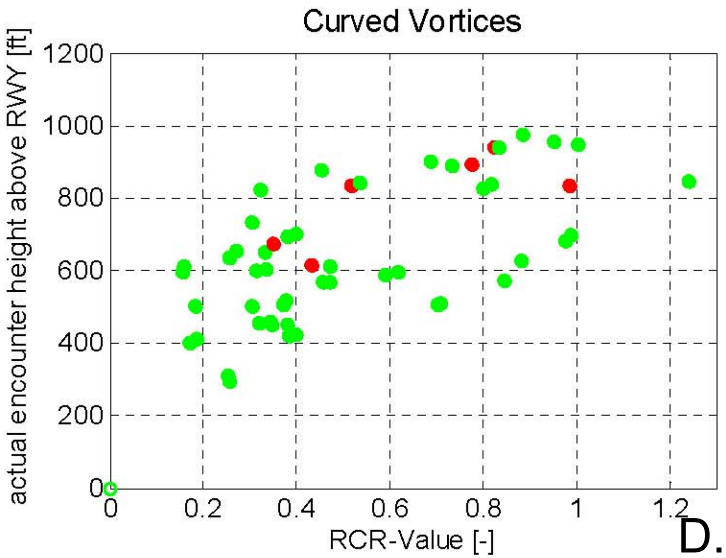
Flight Simulator Study (DLR-FT)



N*=0
ε*=0.23
t*=3.5

$$RCR = \frac{CR_{WV}}{CR_{a/c}}$$





Flown manually

D. Vechtel

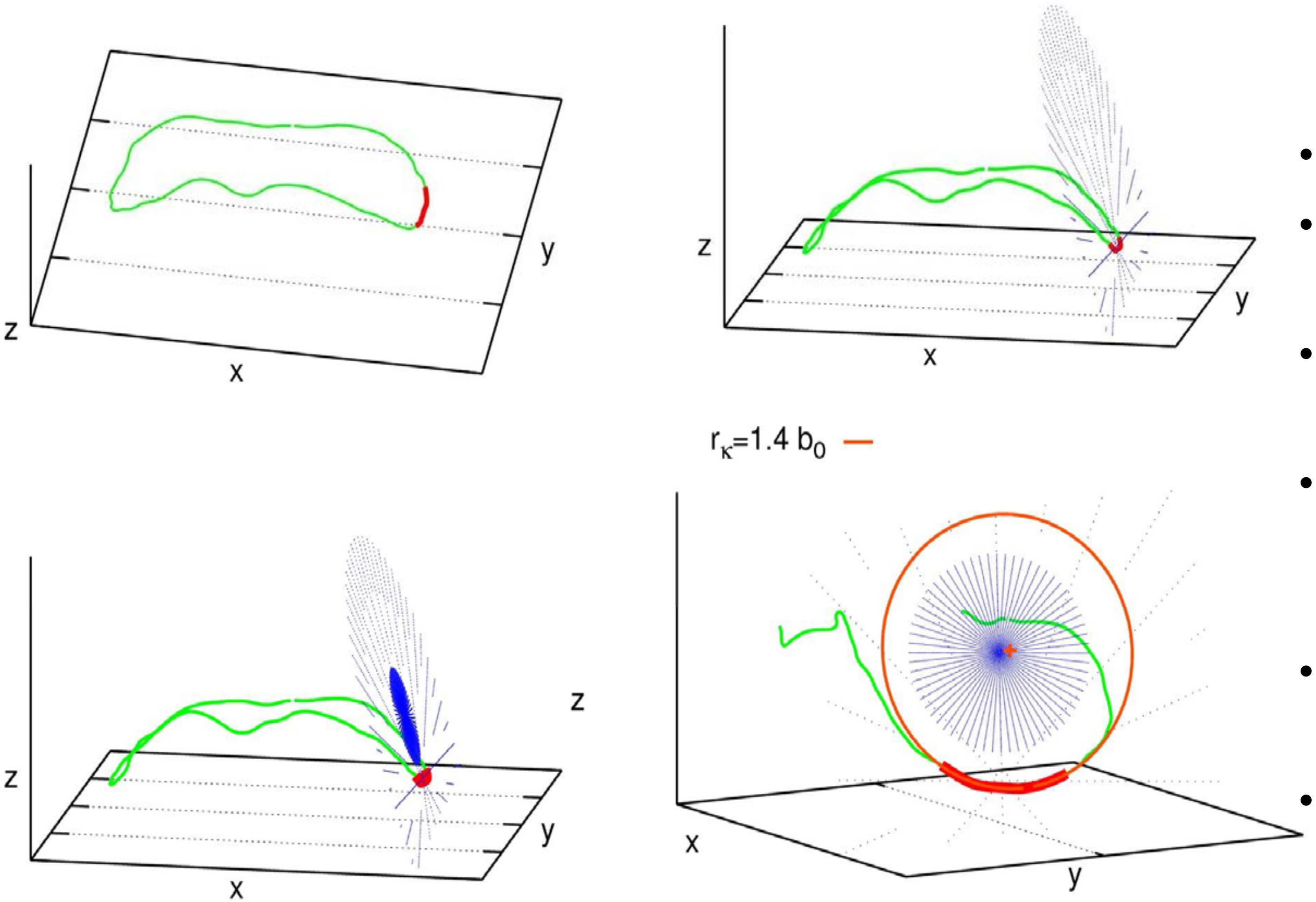
Summary & Outlook

- P2P as core module of wake vortex prediction system (WSVBS)
- Decay of circulation due to N*:
  - Decay caused by baroclinic vorticity
- Decay of circulation due to ε*:
  - Decay enhanced by the formation of vortex rings
  - Vortex rings are often stable and long-living
  - Vortex evolution phases:
    diffusion phase, phase of rapid decay, ring diffusion phase
- P2P → P3P
- C_r important also several t_0 later

→ Colleagues from DLR Braunschweig investigate encounters with curved vortices in detail

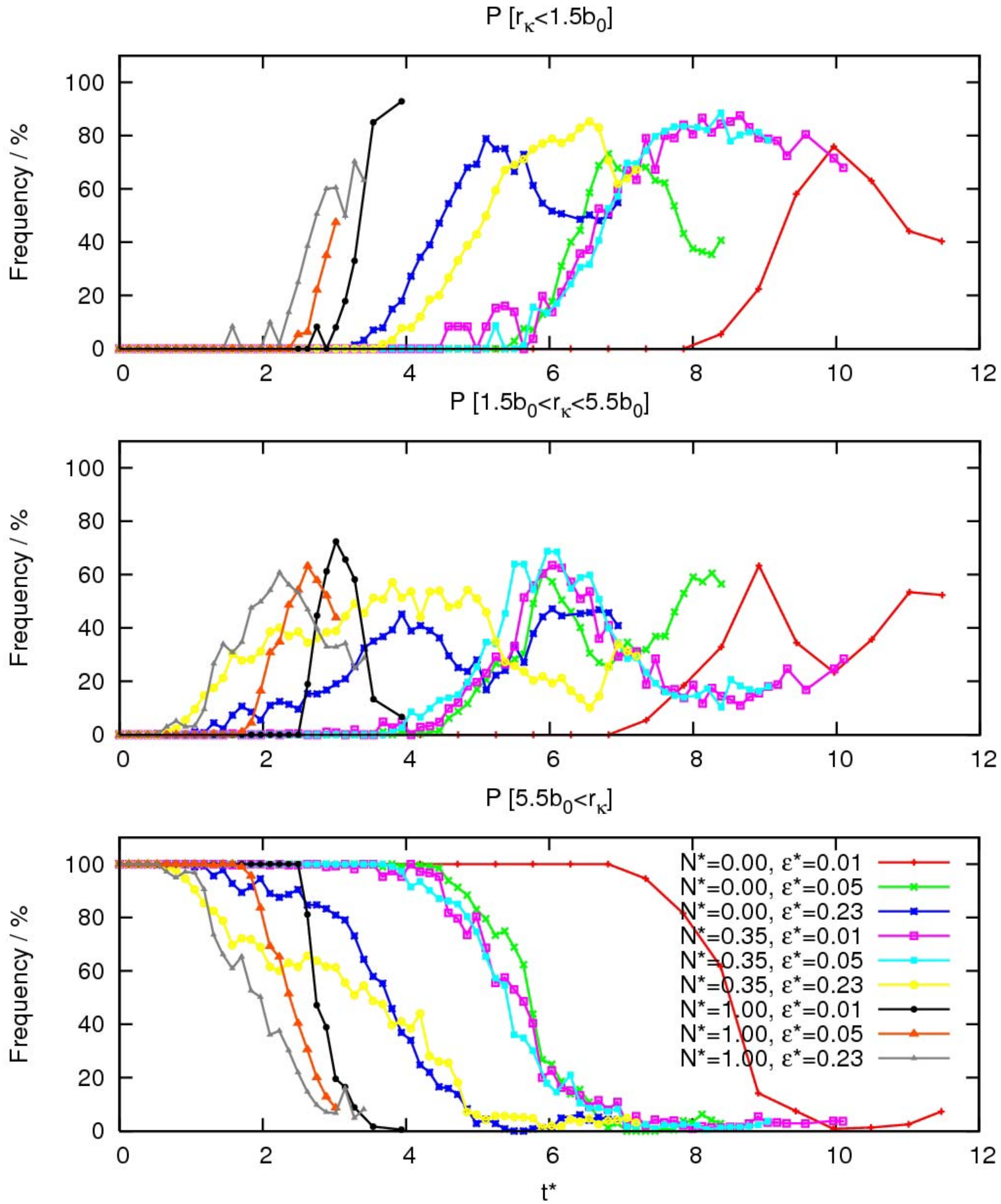

Thanks for your attention!

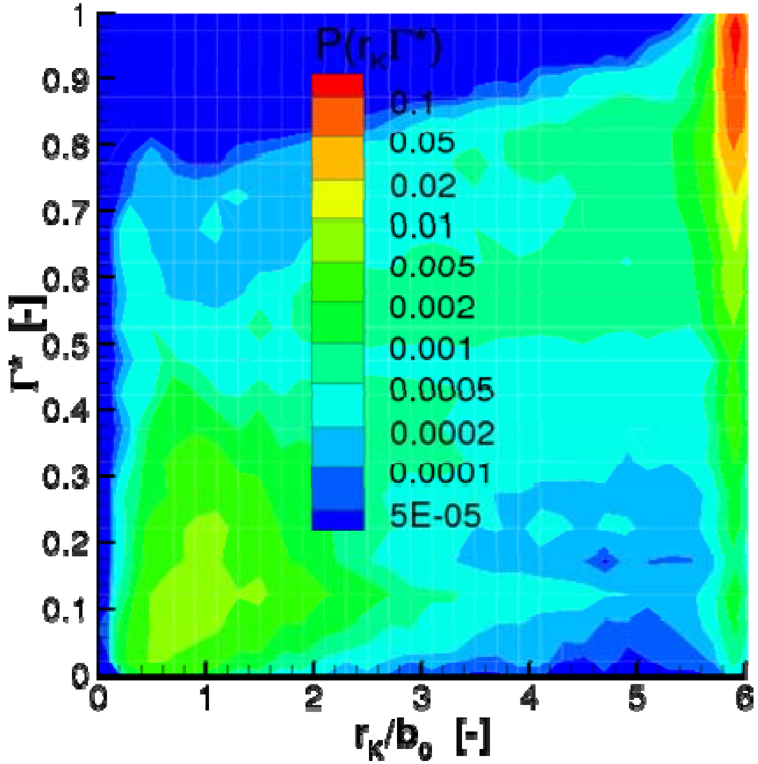
Vortex Curvature Detection



- $l=1.5b_0$
- Quasi-2D assumption
- Star shaped search pattern
- Optimization via method of least squares
- Coarse and fine search pattern
- $r_{\kappa,max}=6b_0$

Results Curvature Radii

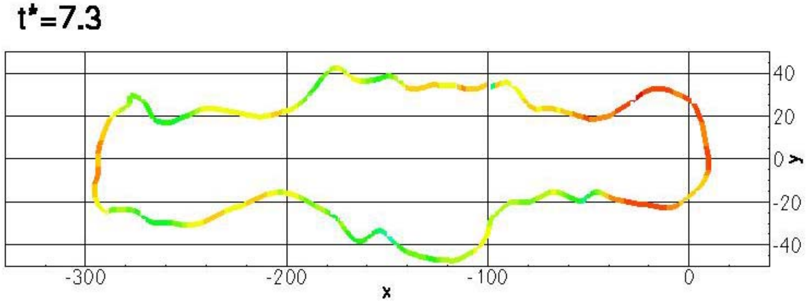




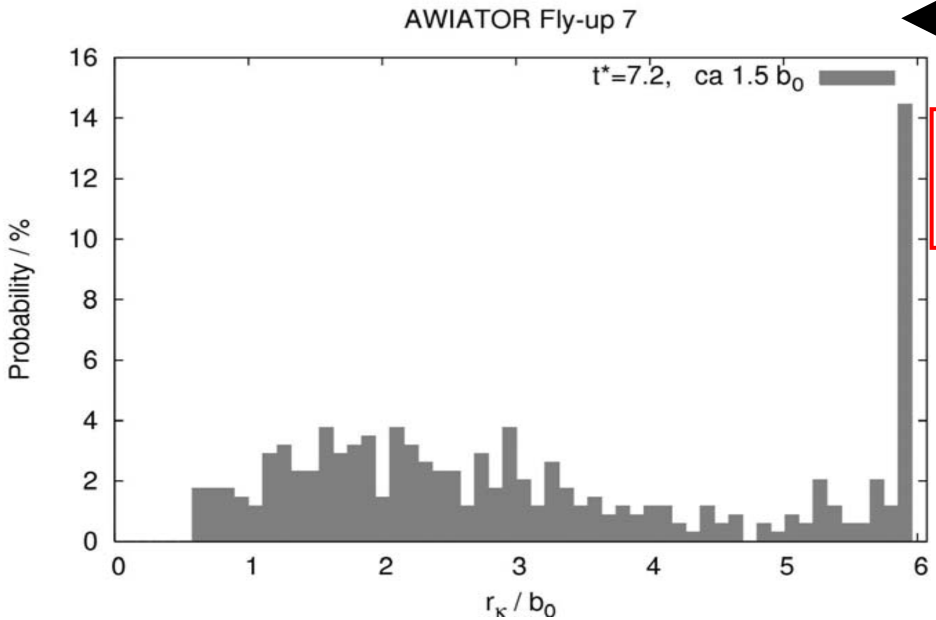
Validation AWIATOR

t*=7.2 **AWIATOR Fly-up 7: N*=0.28, ε*=0.06**

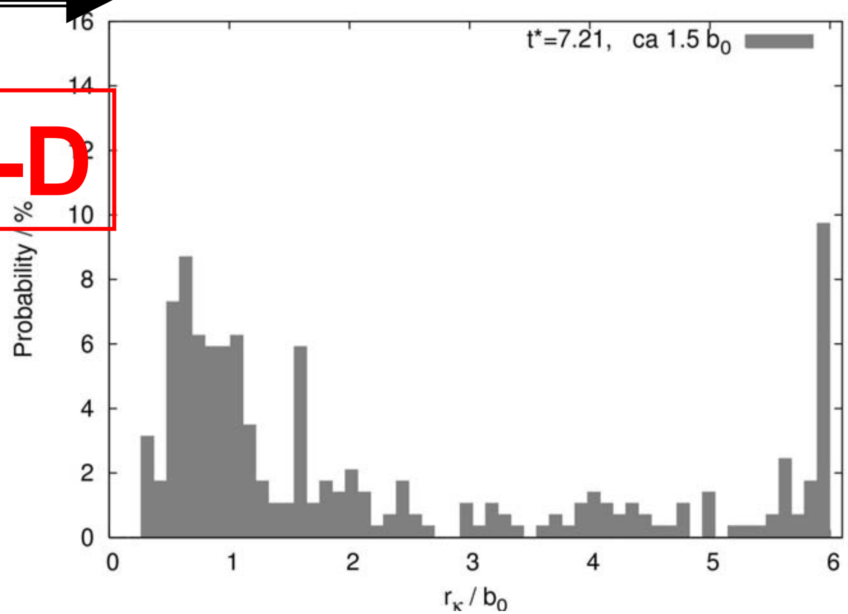
LES: N*=0.35, ε*=0.05



Exp - LES



2-D



Overview on Simulations

- LES code LESTUF
  - 2nd order space / time
  - Modified Smagorinsky closure
  - Periodic boundary conditions

  Normalizations based on Γ_0 and b_0

$$t_0 = \frac{b_0}{w_0} \approx 30s\,|_{A340}$$

- Temperature
  - Normalized stratification N* (Brunt-Väisälä-Frequency)
- Turbulence
  - Normalized dissipation rate ε*
  - Easier to obtain than TKE

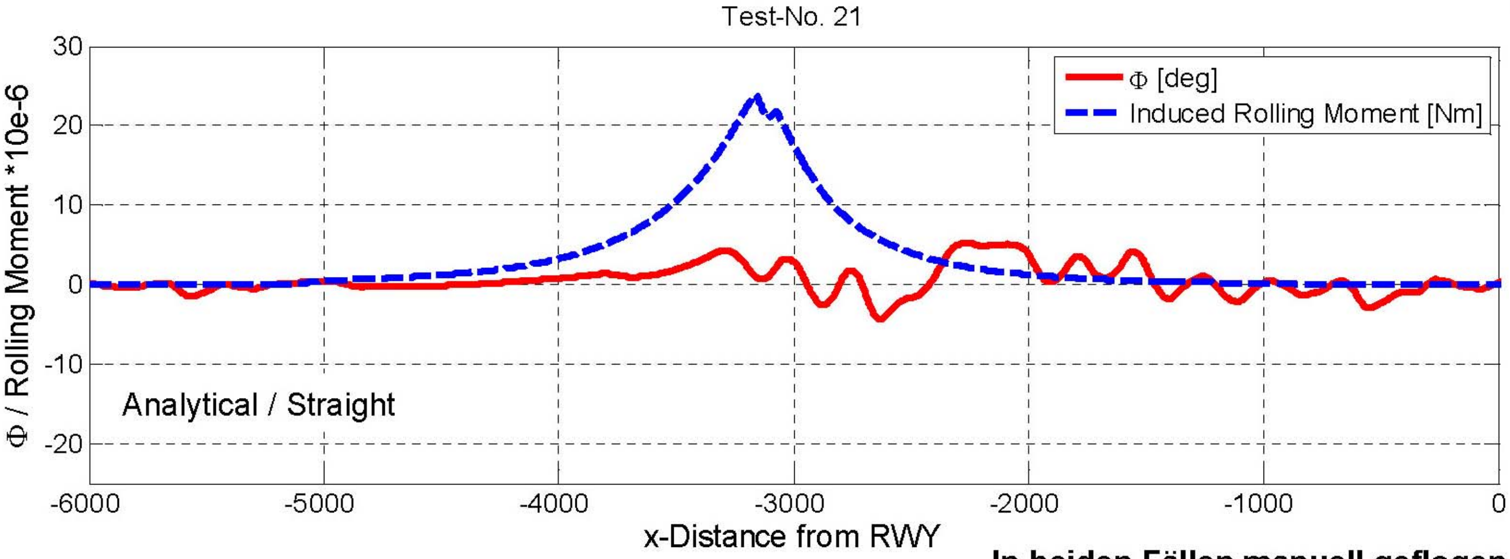
Life Time / t_0		ε*		
		0.01	0.05	0.23
N*	0	>12	>8	>7
	0.35	10	8	8
	1	4	3	3

Until undistinguishable from ambient turbulence

$$N^* = \sqrt{\frac{g}{\theta_0}\frac{\partial\theta}{\partial z}}\,t_0$$

$$\varepsilon^* = \frac{(\varepsilon\, b_0)^{1/3}}{w_0}$$

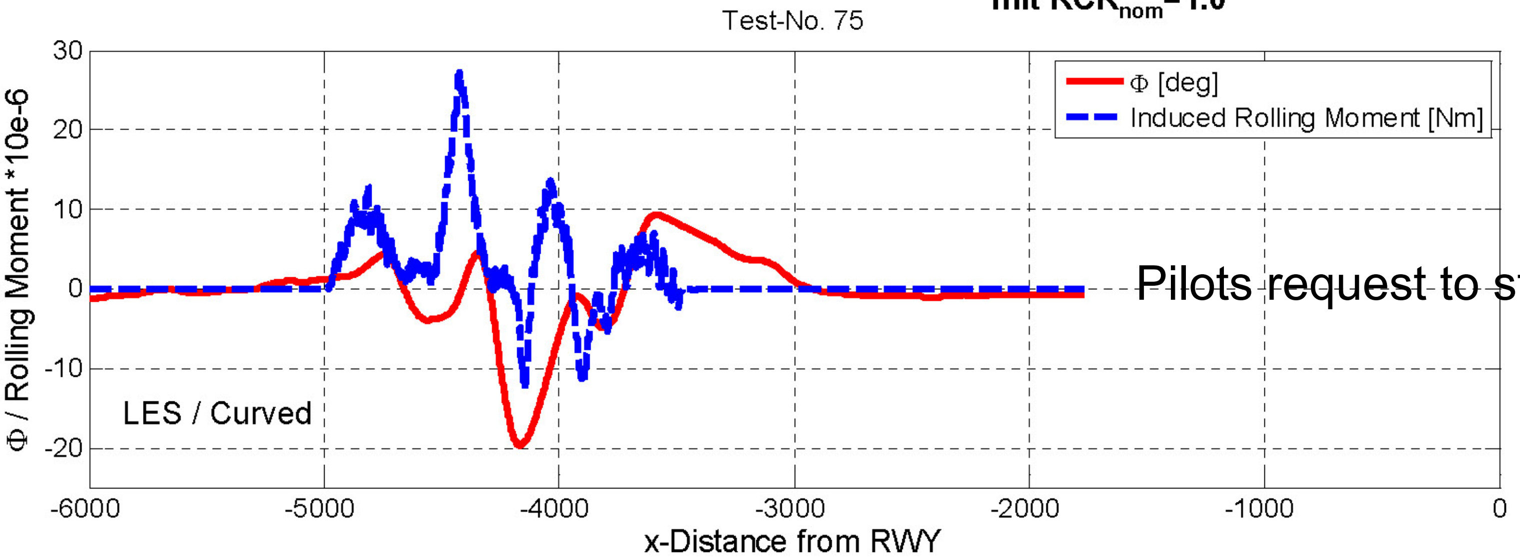
Flight simulator study (3)



N*=0
ε*=0.23
t*=3.5

In beiden Fällen manuell geflogen mit RCR_{nom}=1.0

$$RCR = \frac{CR_{WV}}{CR_{a/c}}$$



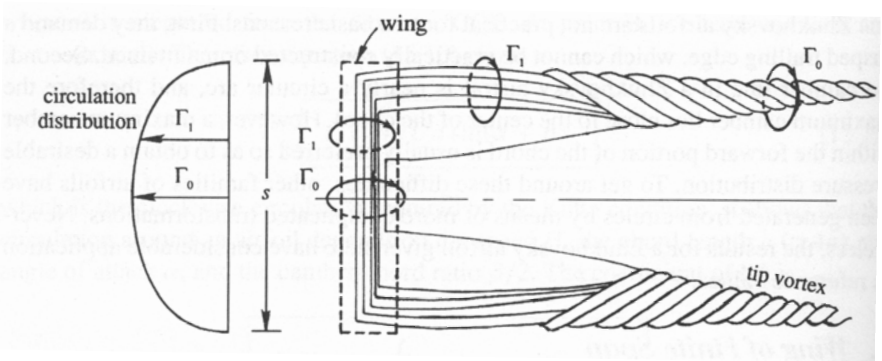
Pilots request to stop

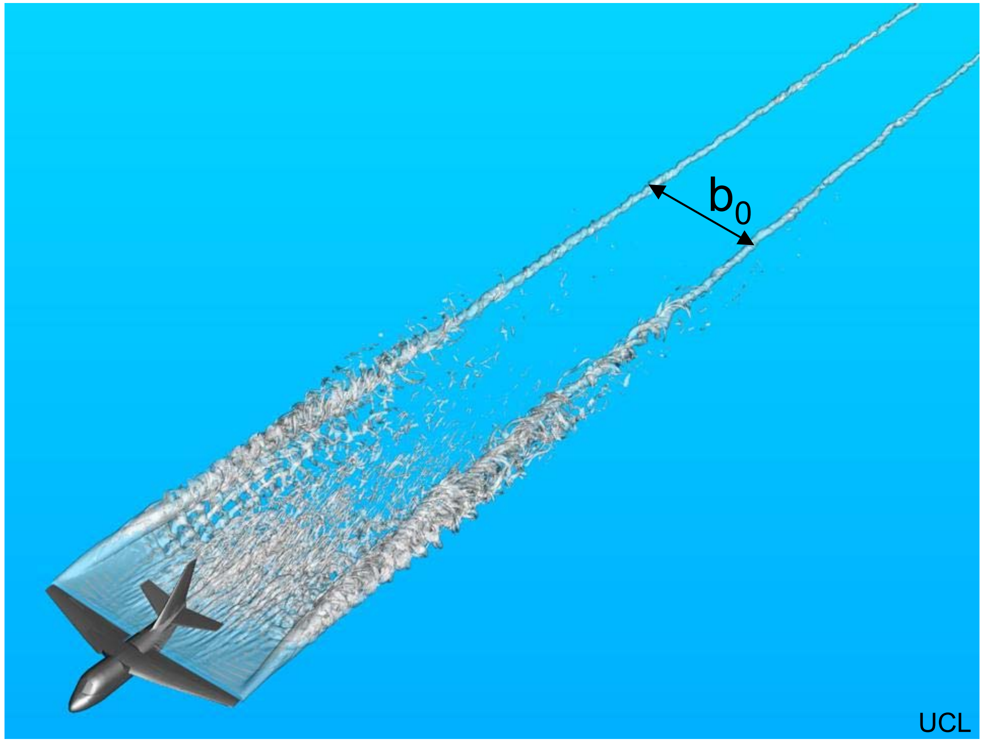
Origin & Roll-up of Wake Vortices

<u>Wake vortices are a direct consequence of lift</u>

$$\boxed{Mg/b_0 = \rho \cdot V \cdot \Gamma}$$

Lifting Theorem of Kutta-Joukovsky

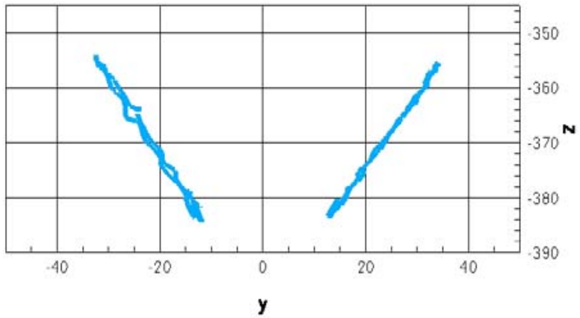




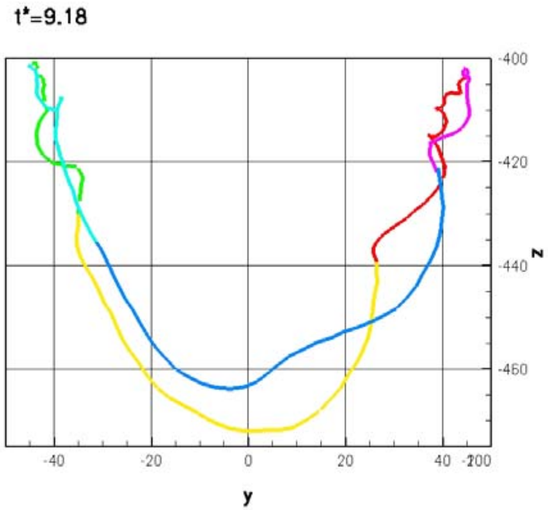
Lifting Line Theory of Prandtl and Lanchester

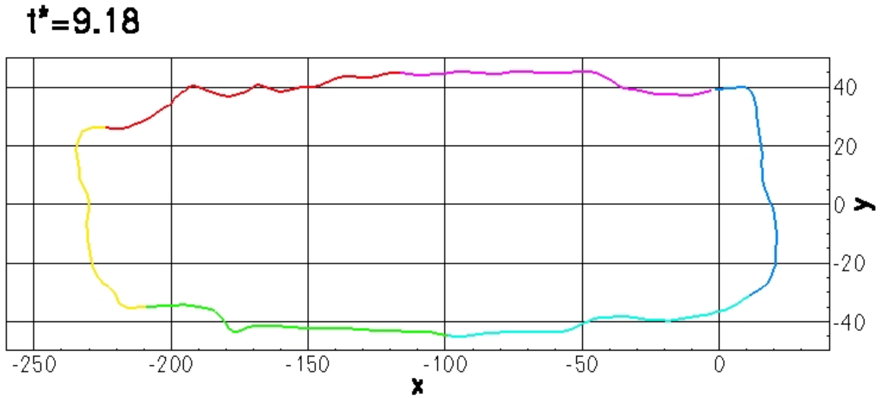
Vortex Curvature Detection

- Goal: realistic curvature radii in aircraft relevant dimensions
- Assumptions:
  - Vortex path divided in segments of approx. 1.5b_0
  - Each segment approx. lies in one plane → quasi-2D
  - Maximal curvature radius 6b_0

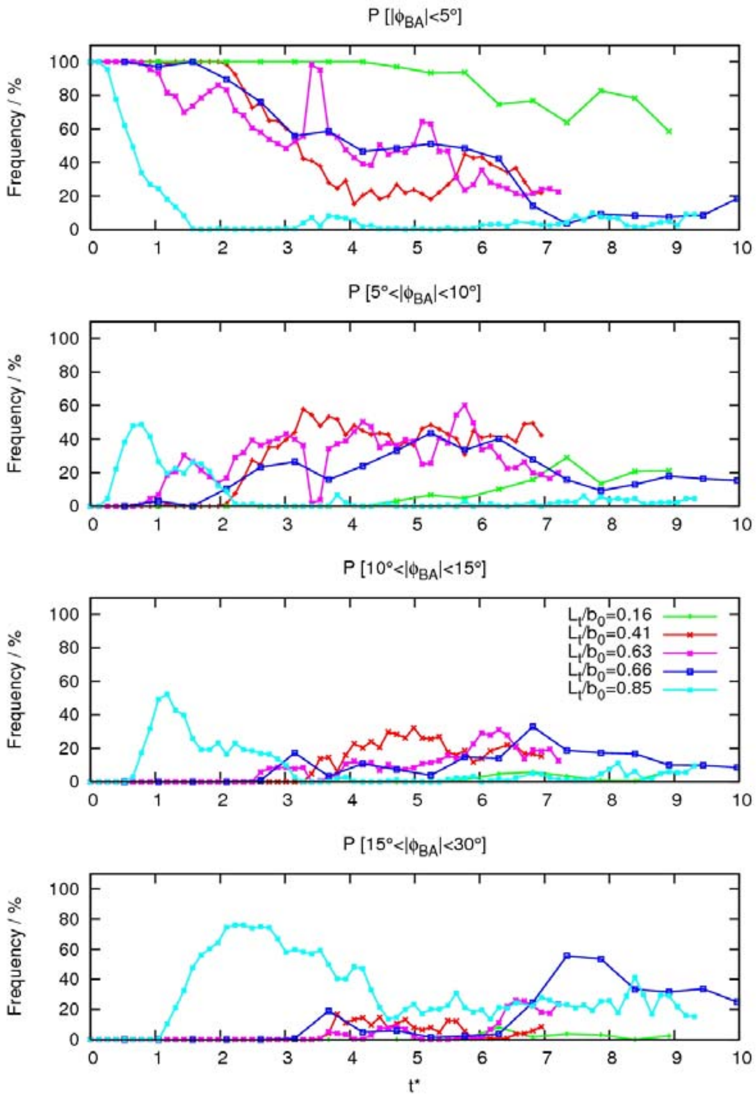


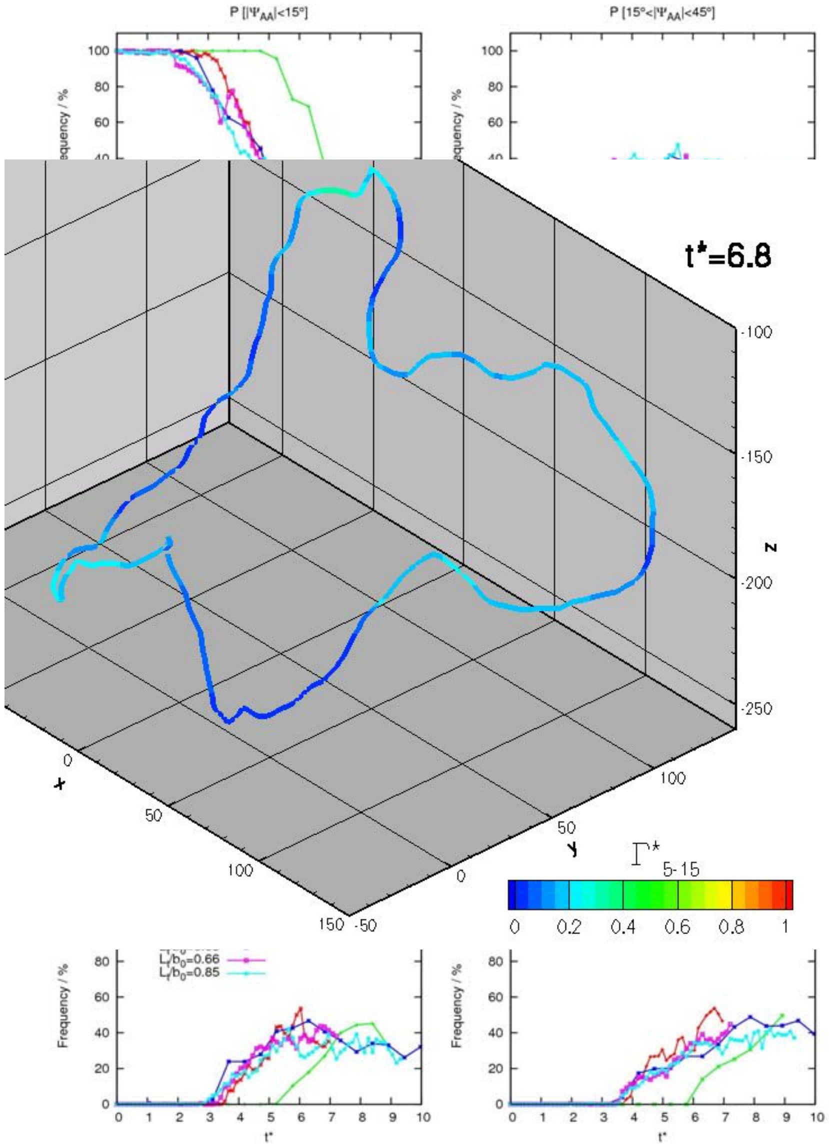
Crow instability theoretically develops in a plane of roughly 48°



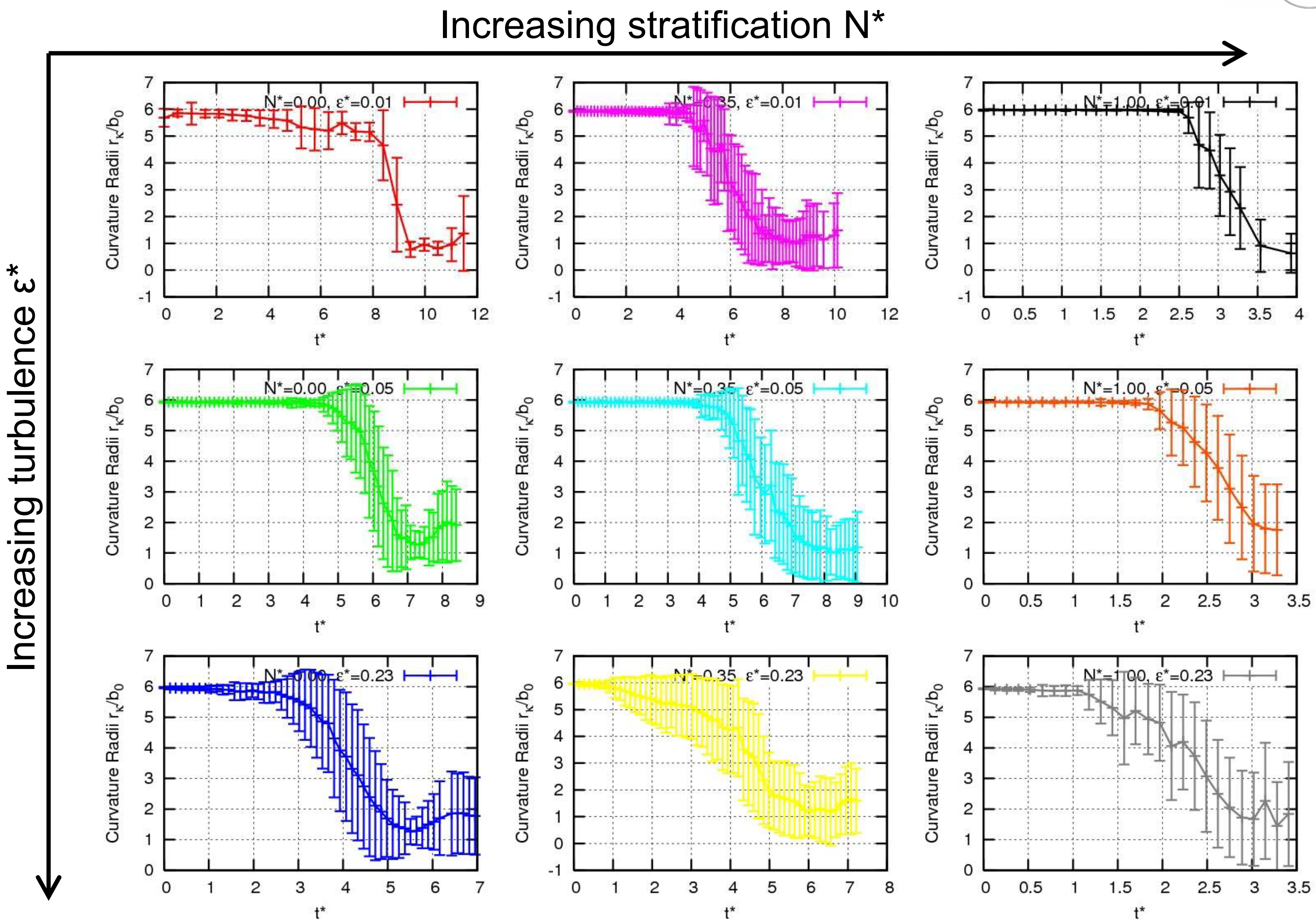


Attitude Angles for varying turbulent length scales



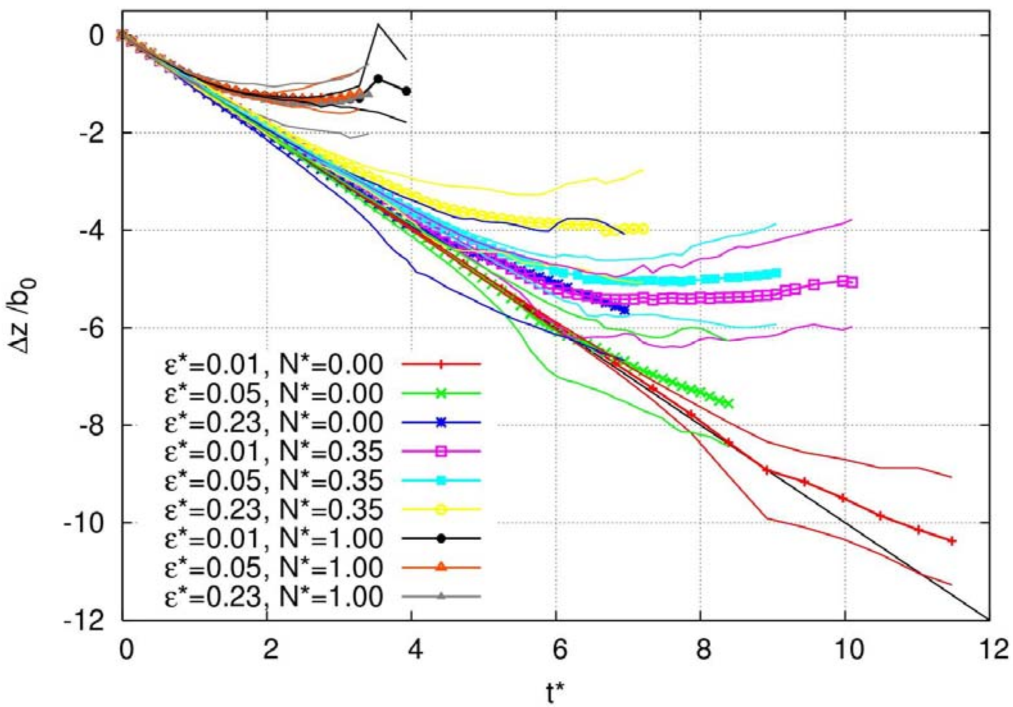


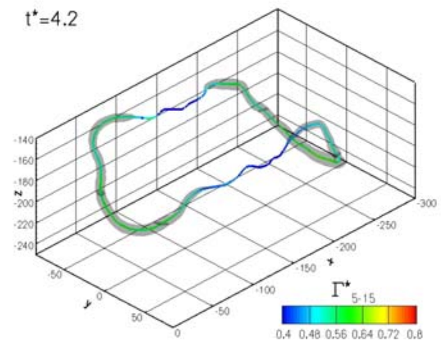
Results Curvature Radii



Results Vortex Position

- During diffusion phase vortices sink as expected
- Sinking rate reduces due to several possible reasons:
  - Sudden loss of circulation during linking
  - Increasing influence of stratification
  - Ground effect (not considered in current investigation)
  - Decreasing mutual velocity induction because of ring shape:
    - Increasing distances of parallel vortex segments
    - After linking circulation orients in span wise direction



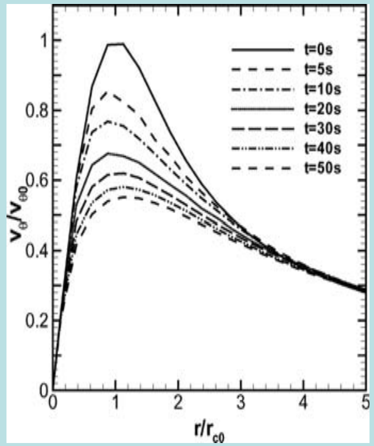


LES code LESTUF (2-1)

- Finite Differences 2nd order on a staggered grid
- Closure Model: Modified Smagorinsky (NaCoo)
- Uses Boussinesq-Approximation
- Periodic boundary conditions
- Allows pre-runs for turbulence spectrum to develop
- Developed at DLR

Pre-run generates fully developed turbulent temperature and velocity field incl. shear and stratification

Set-in of vortex pair (Lamb-Oseen vortex)



Main LES

Post-processing
(vortex track, strength, curvature…)

Boussinesq – Approximation (2-3)

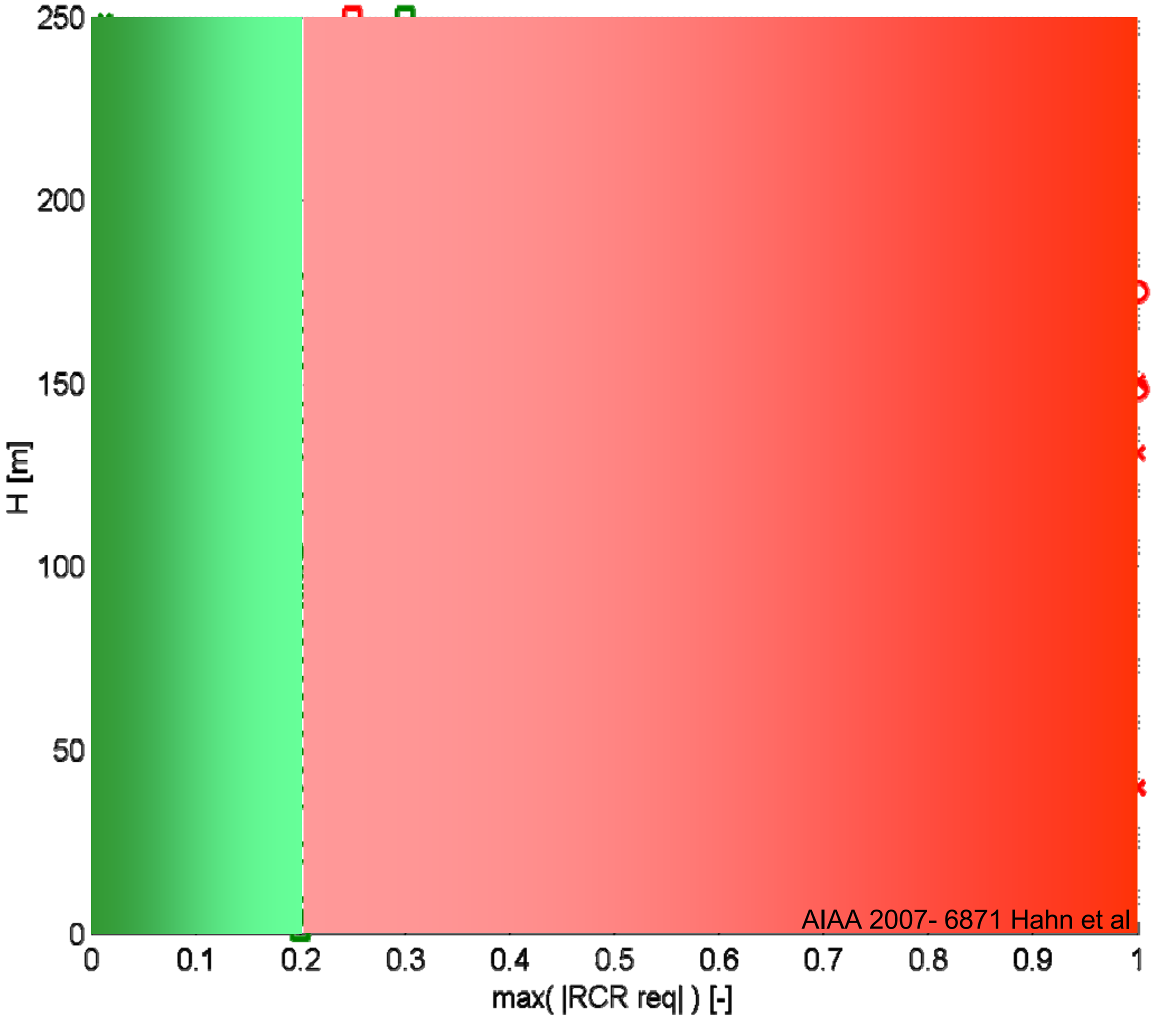
- Baroclinic vorticity production: $\frac{D\omega_{BV}}{Dt} = \frac{1}{\rho^2}\nabla\rho\times\nabla p$
- Boussinesq Definition: $\rho=\rho_0+\rho'$ is replaced by its reference value ρ_0 everywhere except where it multiplies gravity

$$(\rho_0 + \rho')\frac{D\vec{u}}{Dt} = -\nabla p - (\rho_0 + \rho')\vec{g} + \eta(\rho_0 + \rho')\nabla^2\vec{u}$$

$$\frac{\partial}{\partial x} = 0$$

Vorticity formulation	Pressure term	Gravity term
complete	$\nabla\times\left(\frac{-1}{\rho}\cdot\nabla p\right) = .. = \frac{1}{\rho^2}\begin{pmatrix}\frac{\partial\rho}{\partial y}\frac{\partial p}{\partial z} - \frac{\partial p}{\partial y}\frac{\partial\rho}{\partial z}\\ \frac{\partial\rho}{\partial z}\frac{\partial p}{\partial x} - \frac{\partial p}{\partial z}\frac{\partial\rho}{\partial x}\\ \frac{\partial\rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial p}{\partial x}\frac{\partial\rho}{\partial y}\end{pmatrix}$	$\nabla\times(\vec{g}) = \nabla\times(\nabla\Pi) = 0$
Boussinesq	$\nabla\times\left(\frac{-1}{\rho_0}\cdot\nabla p\right) = 0$	$\nabla\times\left(\frac{-\rho'}{\rho_0}\cdot\vec{g}\right) = ... = \frac{1}{\rho_0^2}\left(\frac{\partial\rho'}{\partial y}\frac{\partial p}{\partial z}, -\frac{\partial\rho'}{\partial x}\frac{\partial p}{\partial z}, 0\right)^T$

Risk perception from pilots



Cr,max (B737)= 0.06

	RCR = Cr,WV / Cr,a/c
t*=0	4.2
t*=9.4	1.66

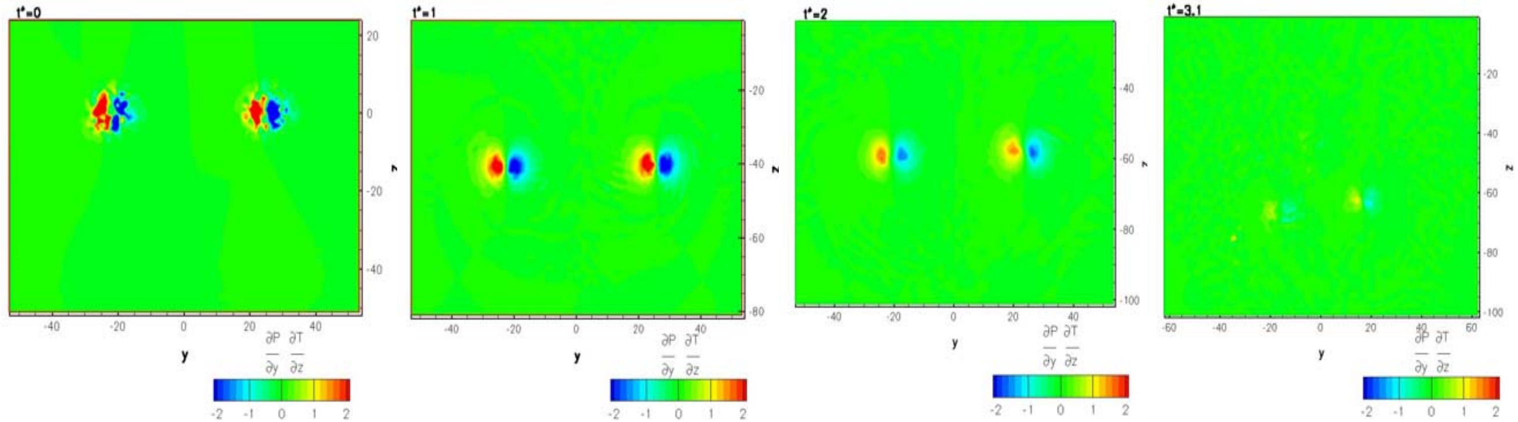
acceptable (95)

unaccetable (19)

Boussinesq – Approximation (2-4)

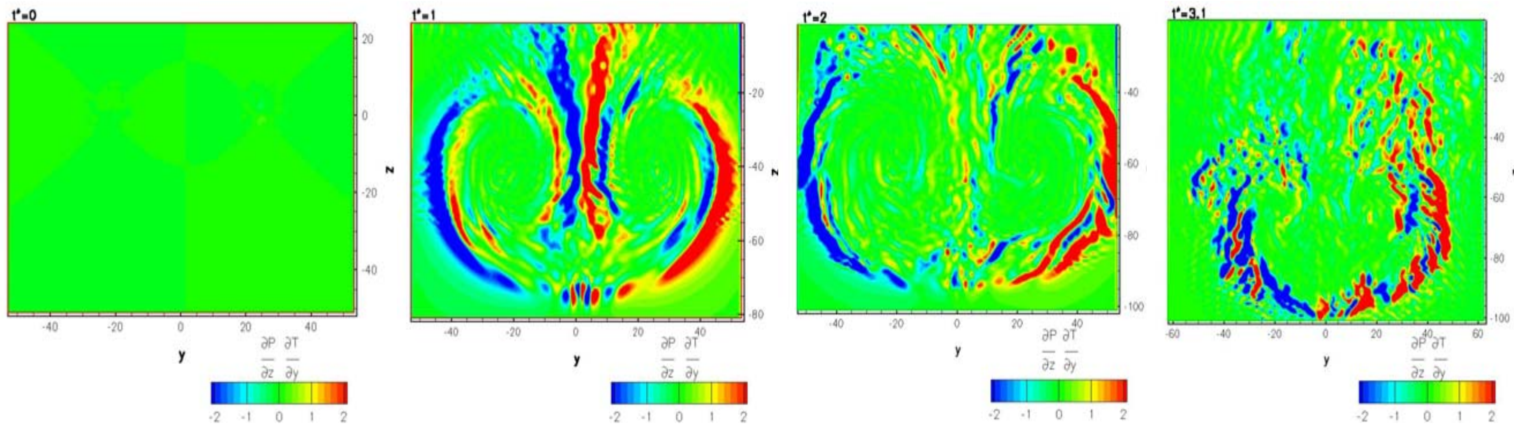
- Example: baroclinic vorticity (BV) for N*=1, ε*=0.01
  - Not covered by Boussinesq-Approximation

$$-\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial z}$$

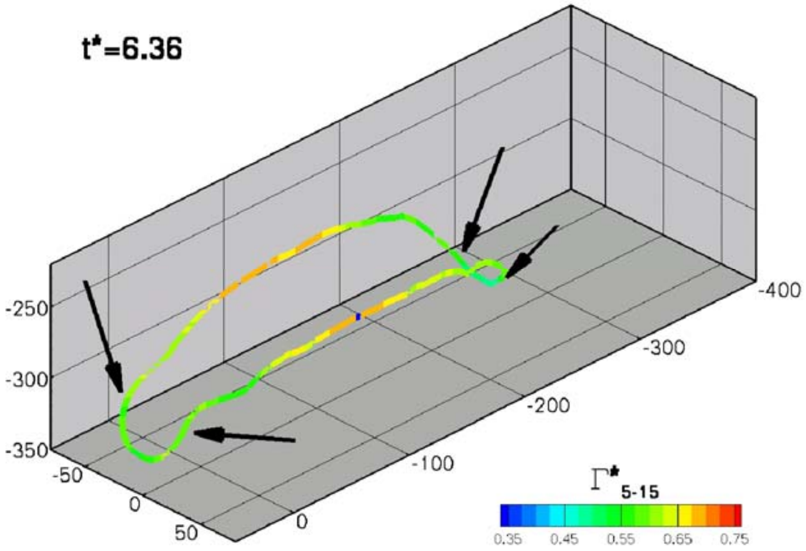


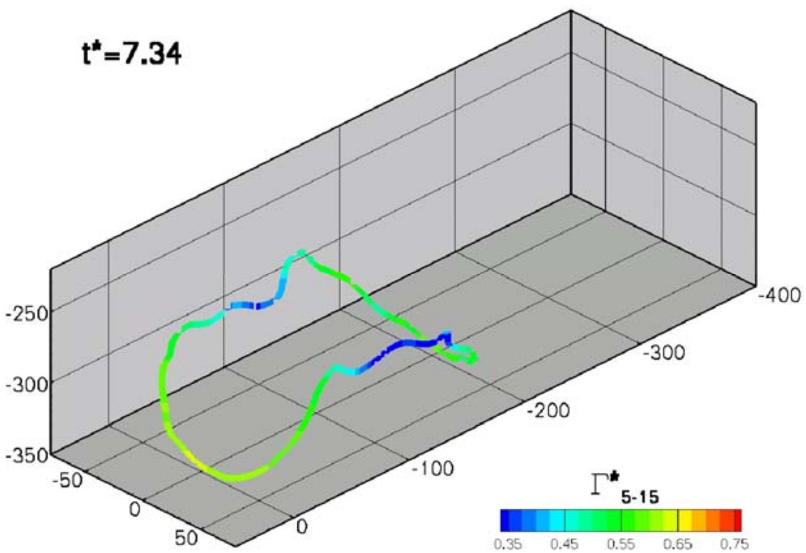
  - covered by Boussinesq-Approximation

$$-\frac{\partial p}{\partial z}\frac{\partial \rho}{\partial y}$$

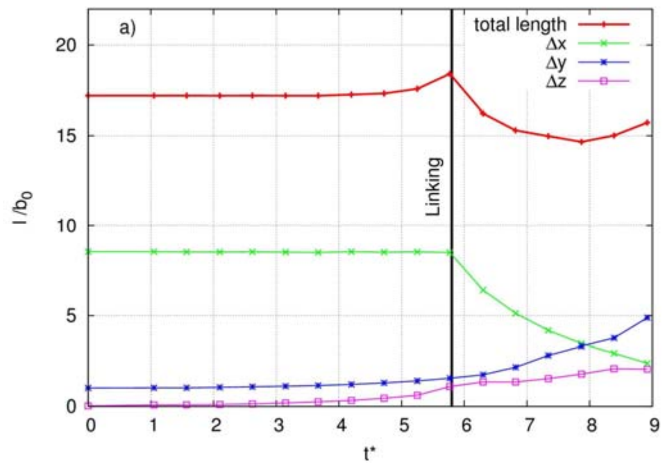


Change of Circulation (4-3)



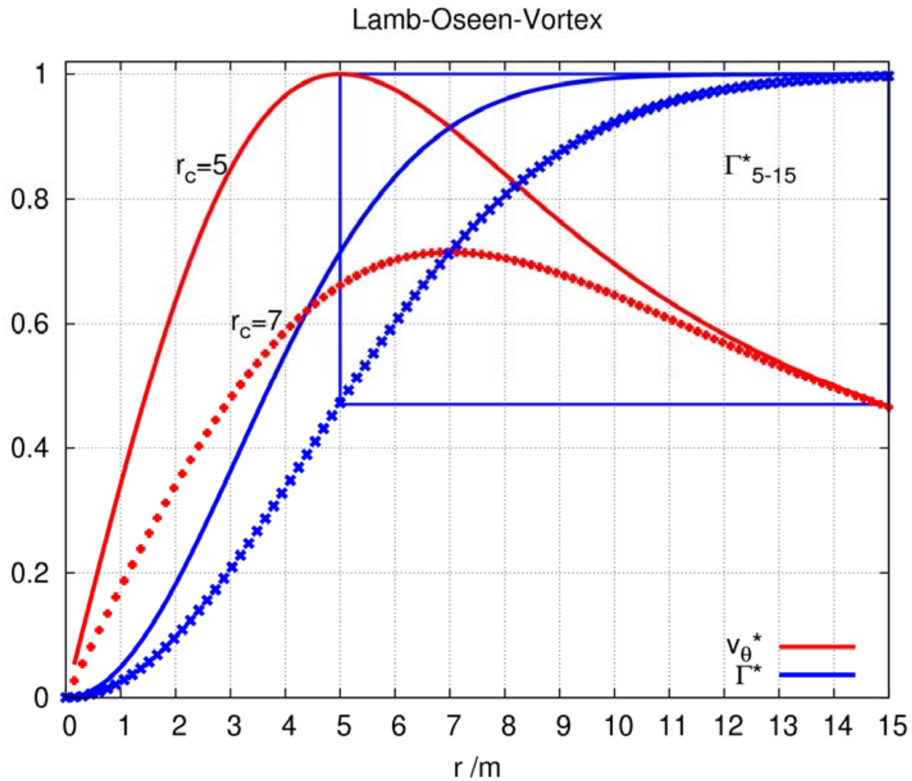


- Circulation change away from flight direction segments
  → old method misses this tendency
  → observed in experiments (Delisi et al.)
- Change of vortex path topology due to mutual velocity induction of vertically oriented vortex elements



Change of Circulation (4-4)

Conservation of mass and momentum $l \propto \frac{1}{r_c^2}$ $v \cdot r_c = const$



$$V_{\theta,L-O} = \frac{\Gamma_0}{2\pi r}\left(1 - \exp\left(-1.2526\left(\frac{r}{r_c}\right)^2\right)\right)$$

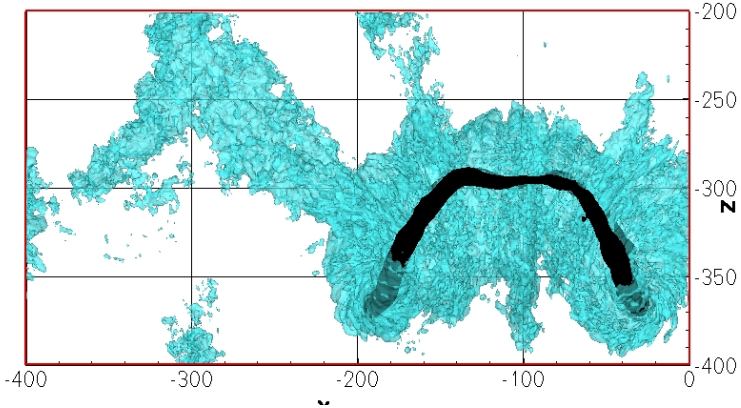
$$\Gamma = \int_0^{2\pi} V_\theta r d\theta$$

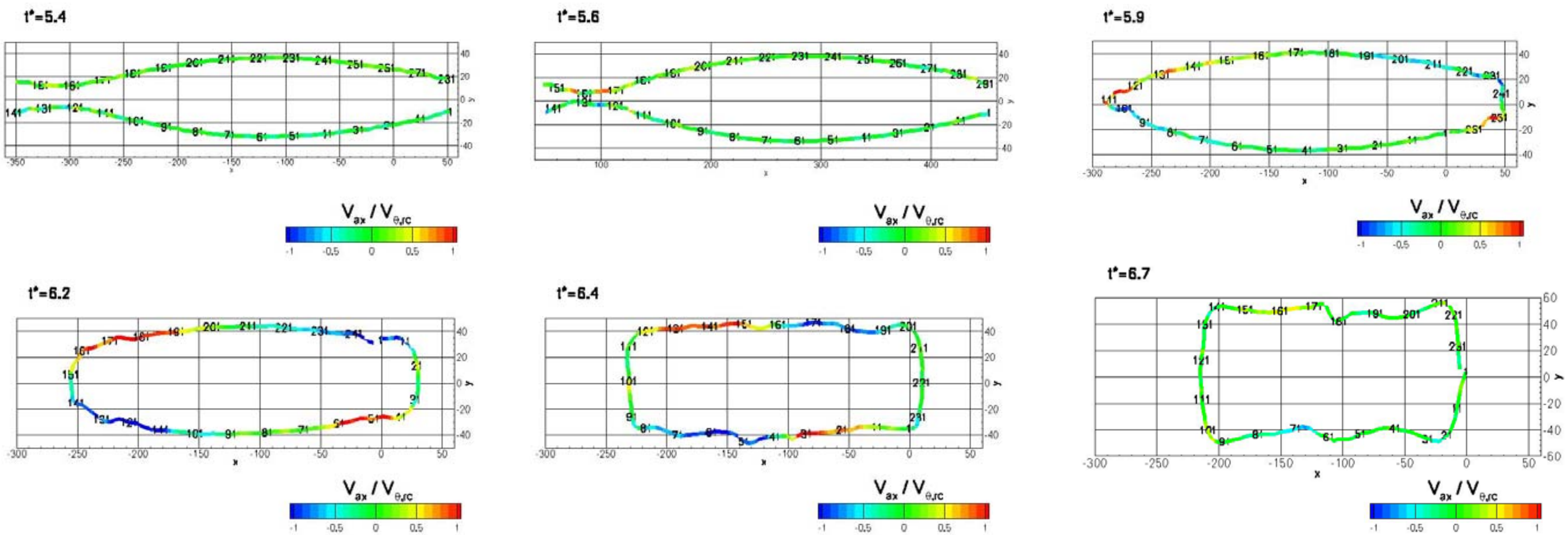
Helmholtz theorem is not violated!

Additional Contribution to loss of circulation in flight direction (4-5)

- Axial velocities within vortex tubes are intensified by vortex linking, because
  - Linking starts with cancellation of vorticity
  - Reduced maximal tangential velocities
  - Pressure deficit locally reduced
  - Pressure perturbations propagate at approx. $v_{pp} \approx v_{\theta,max}$
- Collision of pressure waves produces vortex bursting (Moet et al)

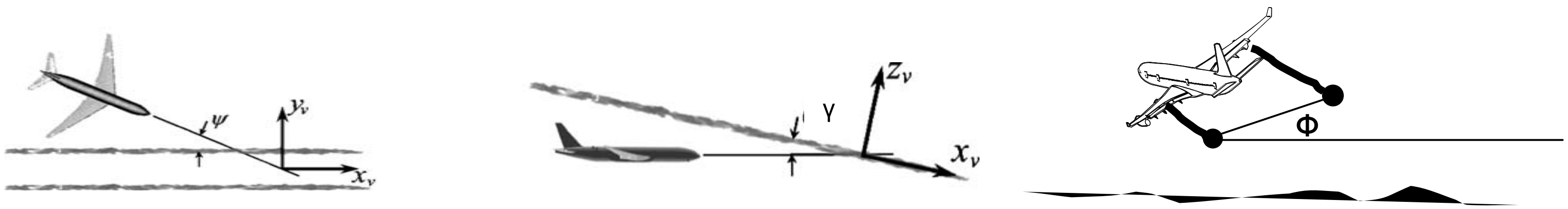
t*=7.3

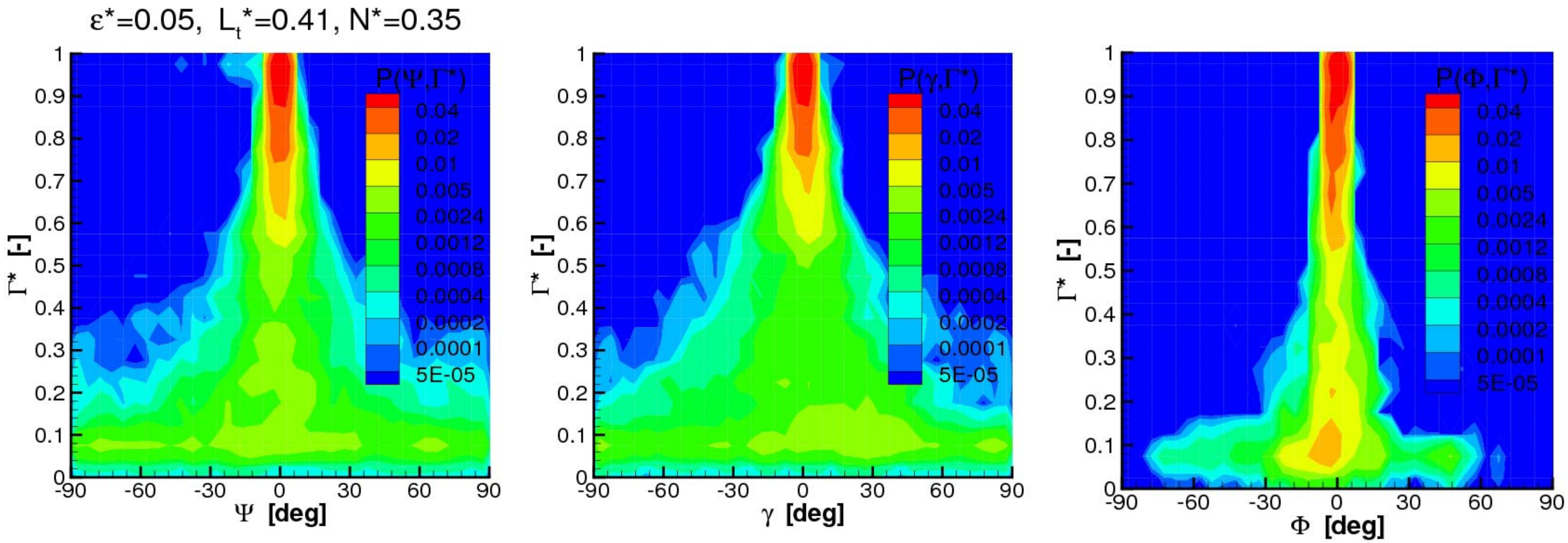




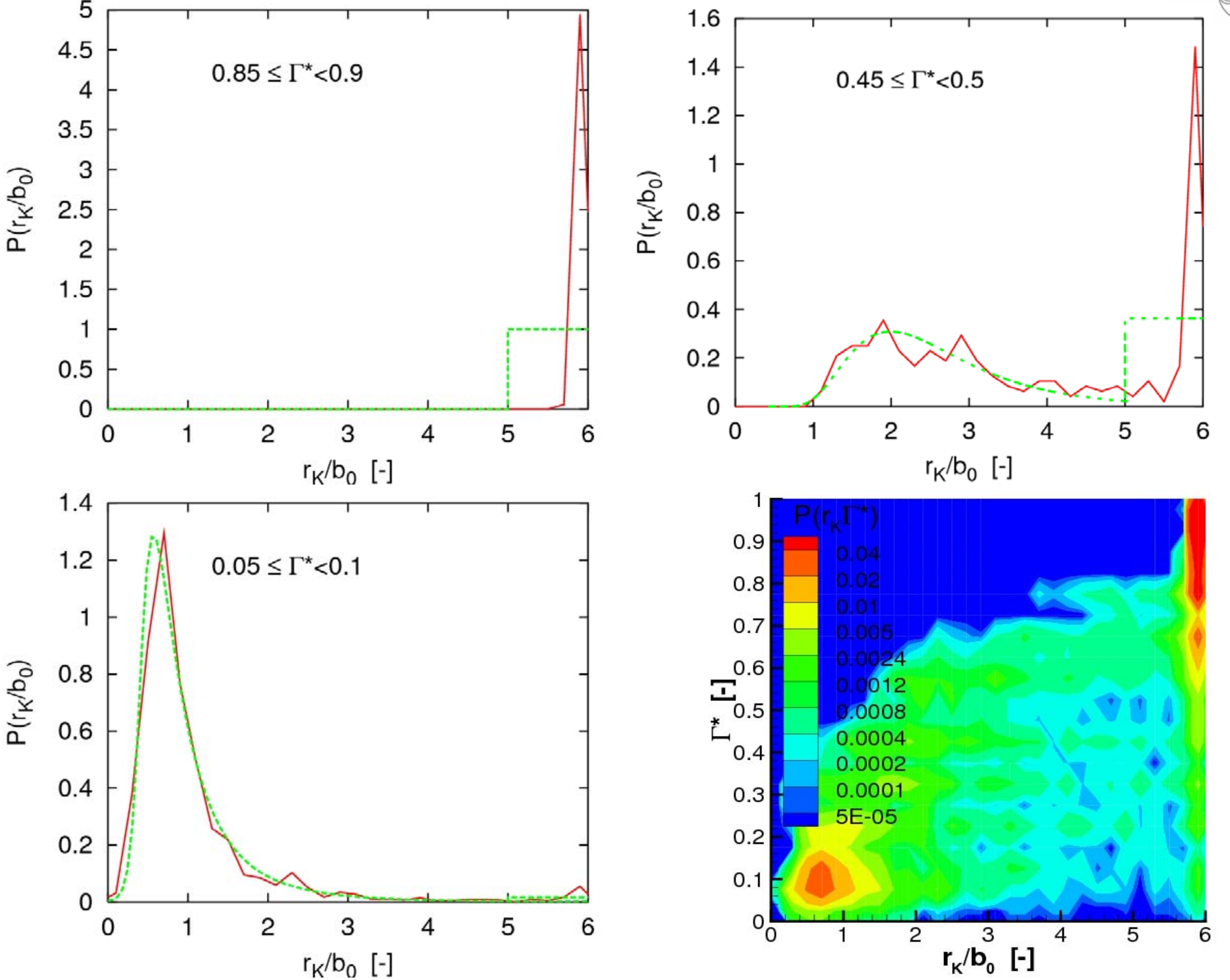
Results for P2P (Attitude Angles)

Attitude angles of one LES with moderate turbulence and stratification

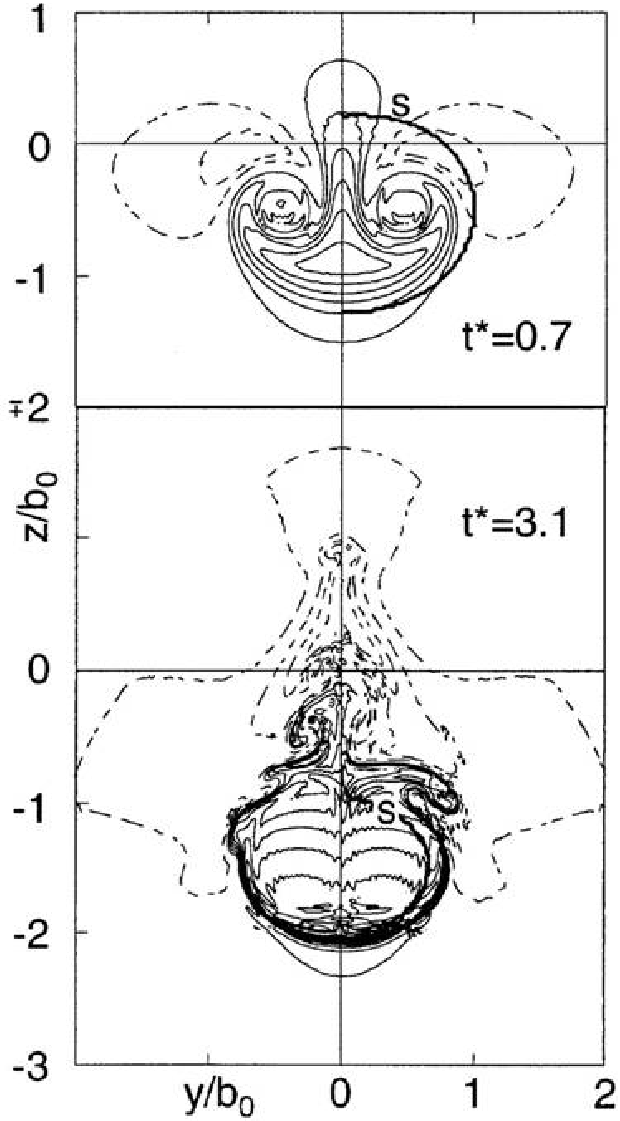




Results in P2P (Curvature Radii)

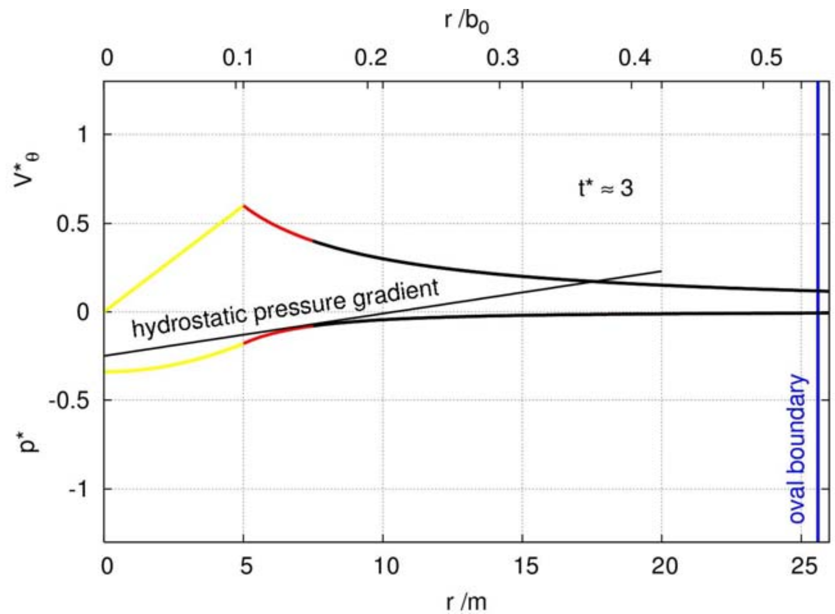


LESTUF (Boussinesq-Approx. -- - Consequences)



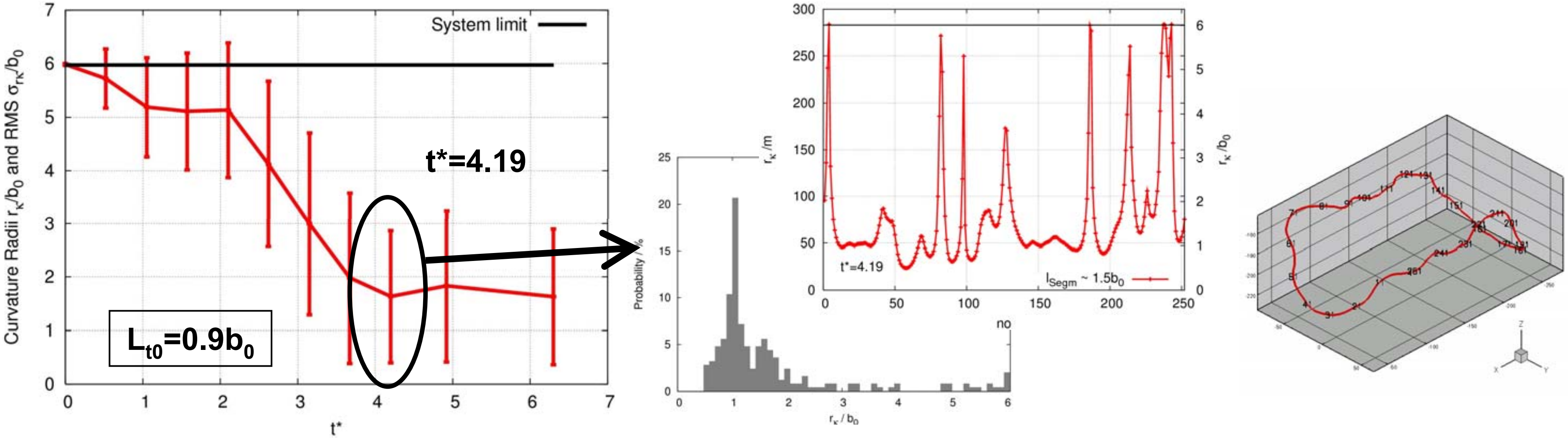
0.5K-iso-therms from Holzäpfel et al

$$-\frac{\partial p}{\partial y}\frac{\partial \rho}{\partial z}$$



- Important pressure gradients of **vortex** only close to the core radius
- t*=0.7: no additional BV
- t*=3.1: development of some new BV, but locally restricted
- Additional BV weak compared to hydrostatic BV

Results (Curvature Radii)



Similar dominant curvature radii!